INTRODUCTION TO DISCRETE MATHEMATICS TEST #3, 23RD JAN 2025

1. Consider the set $H = \{1, 3, 7, 9\}$ with the operation $x \circ y$ defined as the last decimal digit of the number $7xy$.

   i) Compute $(1 \circ 1) \circ (1 \circ 1)$. ANS: 3

   ii) Find the neutral element of $\circ$. ANS: 3

   iii) Find the inverse element 9^{-1}..ANS: 1

2. Let Y be the set of all integers between 300 and 999 (including 300 and 999 themselves) that do not contain the digit 8.

   i) How many elements does Y have? ANS: $6 \cdot 9 \cdot 9 = 486$

   ii) How many elements of Y have three odd digits? ANS: $4 \cdot 5 \cdot 5 = 100$

   iii) How many elements of Y have three different digits? ANS: $6 \cdot 8 \cdot 7 = 336$

3. There are 11 blocks in a bag. They have the following letters on them: A A C C C D D D D R R. We choose 4 blocks from the bag, and as we know there are $\binom{11}{4} = 330$ ways of doing that, ie. 330 four-element subsets. In how many of those 330 sets are there

   enough blocks to form the word CARD? ANS: $3 \cdot 2 \cdot 2 \cdot 4 = 48$

   enough blocks to form the word CAR? CORRECT ANS: 48+12+12+6=72. THE OTHER PLAUSIBLE ANSWER $3 \cdot 2 \cdot 2 \cdot 8 = 96$ IS WRONG.

4* (questions for extra points) Is the set H with the operation $\circ$ described in Problem 1 above a group? ANS: YES.