EIDMA. PROBLEM SET 1

1. Let the symbol $[n]$ denote $\{1, \ldots, n\}$, ie. the set of n smallest positive integers. Let A, B, C be the subsets of $X = [100]$ consisting of all the elements of X divisible respectively by 2, 3 and 5 (eg. $B = \{3, 6, 9, \ldots, 96, 99\}$).

   i) Draw the **Venn diagram** depicting the sets A, B, C and write on it the cardinalities (numbers of elements) of all visible subsets (eg. $A \cap B \cap C$).

   ii) How many elements of X are divisible by exactly one of the numbers 2, 3 and 5? By exactly two? By none of them?

   iii) For how many elements $m \in X$ is the following statement true: 'm is divisible by 2 or by exactly one of 3 and 5'?

2. Using the appropriate truth tables (or some other methods) check that the following sentences are tautologies (ie. they are true for all possible values of all propositional variables).

   i) $p \vee \sim p$; $p \Rightarrow (p \Rightarrow p)$; $p \Leftrightarrow (p \wedge p)$; $\sim (p \wedge \sim p)$

   ii) $(p \wedge q) \Leftrightarrow (q \wedge p)$; $(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$; $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$; $p \Rightarrow (q \Rightarrow p)$; $\sim (p \wedge q) \Leftrightarrow (\sim p \vee \sim q)$; $\sim (p \vee q) \Leftrightarrow (\sim p \wedge \sim q)$

   iii) $(p \vee (q \vee r)) \Leftrightarrow ((p \vee q) \vee r)$; $(p \wedge (q \vee r)) \Leftrightarrow ((p \wedge q) \vee (p \wedge r))$; $(p \vee (q \wedge r)) \Leftrightarrow ((p \vee q) \wedge (p \vee r))$; $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$

   iv) $((p \Leftrightarrow q) \Leftrightarrow (r \Leftrightarrow s)) \Leftrightarrow ((p \Leftrightarrow r) \Leftrightarrow (q \Leftrightarrow s))$

3. Observe that the polynomial $f(p, q) = p + q - pq$ satisfies $f(0, 0) = 0$ and $f(0, 1) = f(1, 0) = f(1, 1) = 1$. Consequently, it coincides with the usual definition of disjunction $p \vee q$.

   i) Find similar polynomials defining conjunction $p \wedge q$, implication $p \Rightarrow q$, equivalence $p \Leftrightarrow q$, and negation $\sim p$.

   ii) Using the polynomials obtained above show that $\sim q \Rightarrow \sim p$ and $p \Rightarrow q$ are equivalent.

   iii) Show in the same manner that $p \Rightarrow (q \wedge r)$ and $(p \Rightarrow q) \wedge (p \Rightarrow r)$ are equivalent.