EIDMA. PROBLEM SET 2

1. Beside the usual $\wedge, \vee, \Rightarrow$ and $\Leftrightarrow$ some other binary (ie. $2-$argument) logical operators are occasionally considered, eg. **xor**, **nand**, **nor**.

   i) Write down the truth tables of those operators. Discuss briefly their properties.

   ii) How many binary logical operators are there in total. How many of them are commutative? Can you verbally describe each operator?

   iii) How many $n-$argument logical operators exist?

   iv) How many binary operators would exist if we allowed three truth values (eg. *true*, *false* and *maybe*) rather than the usual two?

2. Because of the tautologies $(p \wedge q) \Leftrightarrow (\sim (\sim p \vee \sim q))$ and $(p \vee q) \Leftrightarrow \Leftrightarrow (\sim (\sim p \wedge \sim q))$, one of $\wedge, \vee$ can be eliminated from every logical formula at the cost of making it longer.

   i) Show that every possible logical formula can be transformed into an equivalent one that uses only $\sim$ and $\wedge$.

   ii) Show that every possible logical formula can be transformed into an equivalent one that uses only $\sim$ and $\Rightarrow$.

   iii) Can every possible logical formula be transformed into an equivalent one that uses only $\sim$ and $\Leftrightarrow$?

   iv) Show that every possible logical formula can be transformed into an equivalent one that uses only one operator symbol, namely **nand**, also known as the **Sheffer stroke** $|$.

   v) What operator(s) other than **nand** have the same property?

3. The usual notation $p \wedge q$ is called the **infix notation** because the operator $\wedge$ is between its arguments. We could also use a **prefix notation** $\wedge(p, q)$ typical for general functions of two arguments.

   i) Observe that we do not actually need brackets in the prefix notation: $p \Rightarrow q \Rightarrow r$ is ambiguous, as it can mean two inequivalent things, namely $(p \Rightarrow q) \Rightarrow r$ or $p \Rightarrow (q \Rightarrow r)$, while $\Rightarrow\Rightarrow pqr$ and $\Rightarrow p \Rightarrow qr$ are clearly different.

   ii) Write a few of your favourite tautologies from PROBLEM SET 1 in the prefix notation.

   iii) Is $\Rightarrow \wedge p \vee qr \vee \wedge pq \vee qr$ a tautology?

   iv) Guess what the **postfix notation** could be and try to use it.