EIDMA. PROBLEM SET 3

1. Write down symbolically the following sentences:

   i) 3536824785 is divisible by two or by three.

   ii) 23 is a prime number.

   iii) The square of every real number is positive.

   iv) No more than two distinct reals can ever have equal squares.

   v) There are infinitely many prime numbers.

   vi) The greatest common divisor of 999 and 2369 equals n.

   vii) There is no greatest negative integer number.

   viii) There is no greatest negative real number.

   ix) Not all odd positive integers are indivisible by four.

   x) Every natural number can be written as the sum of four squares of natural numbers.

   Remark. Apart from the logical operators and quantifiers other well-known mathematical symbols (eg. $+$, $>$, $\in$) may also be used.

2. Using your previous knowledge of mathematics decide if the above sentences are true.

3. Interpret the following sentences by expressing them in non-technical English:

   i) $(\forall x \in \mathbf{N})(\exists y \in \mathbf{N})x < y$

   ii) $(\forall x \in \mathbf{N})(\exists y \in \mathbf{N})x > y$

   iii) $(\exists y \in \mathbf{N})(\forall x \in \mathbf{N})x < y$

   iv) $(\forall x \in \mathbf{N})(\forall y \in \mathbf{N})((x < y) \Rightarrow (\exists z \in \mathbf{N})(x < y \wedge y < z))$

   v) $(\forall x \in \mathbf{R})(\forall y \in \mathbf{R})((x < y) \Rightarrow (\exists z \in \mathbf{R})(x < y \wedge y < z))$

4. Which of the above sentences are true?