EIDMA. PROBLEM SET 4 PART 2

Let us first fix the notation for the open and closed intervals and disks:

$$(a;b) = \{x \in \mathbb{R} : a < x < b\}, \langle a;b\rangle = \{x \in \mathbb{R} : a \leqslant x \leqslant b\},$$

$$\langle a;b) = \{x \in \mathbb{R} : a \leqslant x < b\}, (-\infty;b\rangle = \{x \in \mathbb{R} : x \leqslant b\}, \text{ etc.}$$

$$D((a,b),r) = \{(x,y) \in \mathbb{R}^2 : (x-a)^2 + (y-b)^2 < r^2\},$$

$$\bar{D}((a,b),r) = \{(x,y) \in \mathbb{R}^2 : (x-a)^2 + (y-b)^2 \leqslant r^2\},$$

1. Find the union $\bigcup_{t\in T} A_t$ and intersection $\bigcap_{t\in T} A_t$ of the following families of sets:

   i) $A_t = \langle 1 + \frac{1}{t}; 3 + \frac{2}{t}\rangle$ for $T = \mathbb{N} \setminus \{0\}$

   ii) $A_t = t\mathbb{N} \setminus \{0, t\}$ for $T = \mathbb{N} \setminus \{0, 1\}$

   iii) $A_t = \bar{D}((t,0),1)$ for $T = (0;\infty)$

   iv) $A_t = \bar{D}((t,0),t)$ for $T = (0;\infty)$

   v) $A_t = D((\cos t, \sin t), 2)$ for $T = \mathbb{R}$

   vi) $A_t = D((2\cos t, 2\sin t), 1)$ for $T = (0;\pi)$

   vii) $A_t = \{(x,y) \in \mathbb{R}^2 : y > tx\}$ for $T = \mathbb{N} \setminus \{0\}$

   viii) $A_t = \{(x,y) \in \mathbb{R}^2 : y = x(x-t)\}$ for $T = \mathbb{R}$

2. Let $A_{m,n}$ be the open interval $(m-n; m+n)$. Compute

$$\bigcap_{m=1}^{\infty} \bigcup_{n=1}^{m} A_{m,n}, \bigcap_{m=1}^{\infty} \bigcup_{n=1}^{\infty} A_{m,n}, \bigcup_{n=1}^{\infty} \bigcap_{m=1}^{\infty} A_{m,n}, \bigcup_{m=1}^{\infty} \bigcap_{n=m}^{\infty} A_{m,n}$$