

EIDMA. PROBLEM SET 5

NOTATION: $\mathbb{N} = \{0, 1, \dots\}$ contains 0, $\mathbb{N}_+ = \mathbb{N} \setminus \{0\} = \{1, 2, \dots\}$ does not contain 0, $k\mathbb{Z} = \{\dots, -2k, -k, 0, k, 2k, \dots\}$ is the set of integers divisible by k .

1. Decide which of the below relations R defined on the set X are equivalence relations:

i) $X = \mathbb{Z}, mRn \equiv m - n \in 2\mathbb{Z}$

ii) $X = \mathbb{Z}, mRn \equiv m + n \in 2\mathbb{Z}$

iii) $X = \mathbb{Z}, mRn \equiv$ the numbers m and n are both even or both odd

iv) $X = \mathbb{Z}, mRn \equiv m - n \in 5\mathbb{Z}$

v) $X = \mathbb{Z}, mRn \equiv m + n \in 5\mathbb{Z}$

vi) $X = \mathbb{Z}, mRn \equiv$ the numbers m and n are both divisible by 5 or both indivisible by 5

vii) $X = \mathbb{Z}, mRn \equiv m \leq n + 2021$

viii) $X = \mathbb{Z}, mRn \equiv mn$ is a square number, i.e. $(\exists k \in \mathbb{Z}) mn = k^2$

ix) $X = \mathbb{N}_+, mRn \equiv mn$ is a square number

x) $X = \mathbb{N} \times \mathbb{N}, (k, l)R(m, n) \equiv k + n = l + m$

xi) $X = \mathbb{N} \times \mathbb{N}, (k, l)R(m, n) \equiv kn = lm$

xii) $X = \mathbb{N} \times \mathbb{N}_+, (k, l)R(m, n) \equiv kn = lm$

xiii) $X = \mathbb{Z}, mRn \equiv m^2 - n^2 \in 8\mathbb{Z}$

xiv) $X = \mathbb{Z}, mRn \equiv 3m^2 + 5n^2 \in 8\mathbb{Z}$

xv) $X = \mathbb{Z}, mRn \equiv m^3 + 5n \in 6\mathbb{Z}$

2. If the relation R from the above problem is an equivalence relation, determine and describe its equivalence classes.