EIDMA. PROBLEM SET 6

NOTATION: $\mathbb{N} = \{0, 1, \ldots\}$ contains 0, $\mathbb{N}_+ = \mathbb{N} \setminus \{0\} = \{1, 2, \ldots\}$ does not contain 0, $k\mathbb{Z} = \{\ldots, -2k, -k, 0, k, 2k, \ldots\}$ is the set of integers divisible by k.

1. Decide which of the below relations R defined on the set X are equivalence relations:

   i) $X = \mathbb{Z}, mRn \equiv m - n \in 2\mathbb{Z}$

   ii) $X = \mathbb{Z}, mRn \equiv m + n \in 2\mathbb{Z}$

   iii) $X = \mathbb{Z}, mRn \equiv$ the numbers m and n are both even or both odd

   iv) $X = \mathbb{Z}, mRn \equiv m - n \in 5\mathbb{Z}$

   v) $X = \mathbb{Z}, mRn \equiv m + n \in 5\mathbb{Z}$

   vi) $X = \mathbb{Z}, mRn \equiv$ the numbers m and n are both divisible by 5 or both indivisible by 5

   vii) $X = \mathbb{Z}, mRn \equiv m \leqslant n + 2021$

   viii) $X = \mathbb{Z}, mRn \equiv mn$ is a square number, i.e. $(\exists k \in \mathbb{Z})\ mn = k^2$

   ix) $X = \mathbb{N}_+, mRn \equiv mn$ is a square number

   x) $X = \mathbb{N} \times \mathbb{N}, (k, l)R(m, n) \equiv k + n = l + m$

   xi) $X = \mathbb{N} \times \mathbb{N}, (k, l)R(m, n) \equiv kn = lm$

   xii) $X = \mathbb{N} \times \mathbb{N}_+, (k, l)R(m, n) \equiv kn = lm$

   xiii) $X = \mathbb{Z}, mRn \equiv m^2 - n^2 \in 8\mathbb{Z}$

   xiv) $X = \mathbb{Z}, mRn \equiv 3m^2 + 5n^2 \in 8\mathbb{Z}$

   xv) $X = \mathbb{Z}, mRn \equiv m^3 + 5n \in 6\mathbb{Z}$

2. If the relation R from the above problem is an equivalence relation, determine and describe its equivalence classes.