EIDMA. PROBLEM SET 7

1. Let $f : X \to Y$ and $g : Y \to Z$ be two functions, and let $g \circ f : X \to Z$ be their composition. Prove that

   i) if f and g are both injective then $g \circ f$ is injective;

   ii) if f and g are both surjective then $g \circ f$ is surjective;

   iii) if f and g are both bijective then $g \circ f$ is bijective;

   iv) if $g \circ f$ is injective then f is injective;

   v) if $g \circ f$ is surjective then g is surjective.

2. If f and g are as above, is it possible that

   i) $g \circ f$ is injective and g is not injective;

   ii) $g \circ f$ is surjective and g is not surjective;

   iii) $g \circ f$ and f are both bijective but g is not bijective?

3. Let now $f : X \to Y$ and $g : Y \to X$ be two functions. Recall what it means that f and g are each other's inverse functions. Can you give an example where $(\forall x \in X) g(f(x)) = x$ is true, but $(\forall y \in Y) f(g(y)) = y$ is false?

4. Give an example of a set X and two functions $f, g : X \to X$, for which the equality $f(g(x)) = x$ is true for all $x \in X$, but the equality $g(f(x)) = x$ is false for **exactly one** $x \in X$.