EIDMA. PROBLEM SET 8

1. Assuming that $x \oplus y = x + y + 1$ and $x \otimes y = xy + x + y$ are two binary operations defined for all $x, y \in \mathbb{R}$,

   i) prove that both $\oplus$ and $\otimes$ are commutative;

   ii) prove that both $\oplus$ and $\otimes$ are associative;

   iii) prove that $\otimes$ is distributive with respect to $\oplus$;

   iv) verify if $\oplus$ and $\otimes$ have neutral elements;

   v) decide which numbers $x \in \mathbb{R}$ have their inverse elements with respect to $\oplus$ and $\otimes$, and which do not.

2. For what values of parameters $a, b \in \mathbb{R}$ the operation $x \ddagger y = xy + ax + by + 2$ defined for $x, y \in \mathbb{R}$ is commutative and associative?

3. For $G = \{1, 3, 5, 9\}$ and $x \circ y$ defined as the last decimal digit of the number $xy + 3x + 3y - 4$

   i) show that $\circ$ is a well-defined operation on G and present its table[1];

   ii) find the neutral element of $\circ$;

   iii) prove that $(G, \circ)$ is a group.

4. For $H = \{1, 3, 7, 9\}$ and $x \circ y$ defined as the remainder left over when the number $xy + 4x + 4y$ is divided by 12

   i) show that $\circ$ is a well-defined operation on H and present its table;

   ii) find the neutral element of $\circ$;

   iii) prove that $(H, \circ)$ is a group.

5. Let P be the polygon with vertices $(1, 0), (2, 0), (4, 1), (4, 2), (3, 4), (2, 4), (0, 3), (0, 2)$ taken in that order. Find all isometries of P. What group do they form?

6. Let Q be the polygon with vertices $(1, 0), (2, 0), (4, 2), (4, 3), (3, 4), (2, 4), (0, 2), (0, 1)$ taken in that order. Find all isometries of Q. What group do they form?

7. Which of the groups introduced above in Problems 3–6 are pairwise isomorphic and which are not?

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[1] *table* as in the multiplication table, not the piece of furniture