EIDMA. PROBLEM SET 9

1. Let X be the set of all five-digit numbers that can be obtained by permuting the digits of the number 12558.

   i) How many elements does X contain?

   ii) How many even numbers are there in X?

   iii) How many odd numbers are there in X?

   iv) Calculate the sum of all odd numbers belonging to X?

2. Repeat the above calculations with the number 10558 (remembering that 0 is not a legal first digit of a number).

3. Let X be the set of all ten-letter words[1] that can be obtained by permuting the letters of the word COLLATERAL.

   i) How many elements does X contain?

   ii) How many elements of X begin and end with L?

   iii) How many elements of X contain three neighbouring letters L?

   iv) How many elements of X contain at least two neighbouring letters L?

   v) How many elements of X contain no neighbouring vowel letters (letters A, E, O)?

4. In how many ways can the elements of an eight-element set be divided into four pairs? (there can be different answers depending on when we treat two ways of dividing as different – please formulate clearly your assumptions and calculate accordingly).

5. How many five-card subsets of the set of 52 cards of the usual deck (containing four aces, four kings, four queens, etc.) contain simultaneously at least two aces, at least one king and at least one queen?

6. A football team consists of 23 players: three goalkeepers, eight defenders, seven midfielders and five forwards. How many different match lineups are possible if we assume that a lineup must consist of exactly 11 players and include exactly one goalkeeper, four or five defenders, at least three midfielders and at least one forward?

[1] The word 'word' is used in the broadest sense. A word doesn't have to mean anything.

7. Prove the following identities. If you can, provide multiple proofs for some or all of them (e.g. combinatorial and algebraic).

$$\binom{n}{n-k} = \binom{n}{k}$$

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$$

$$\binom{n+2}{k+2} = \binom{n}{k+2} + 2\binom{n}{k+1} + \binom{n}{k}$$

$$\binom{m+n}{2} = \binom{m}{2} \cdot \binom{n}{0} + \binom{m}{1} \cdot \binom{n}{1} + \binom{m}{0} \cdot \binom{n}{2}$$

$$\binom{n}{3} = \binom{n-1}{2} + \binom{n-2}{2} + \ldots + \binom{3}{2} + \binom{2}{2}$$

$$\binom{2n}{0} + \binom{2n}{2} + \ldots + \binom{2n}{2n} = \binom{2n}{1} + \binom{2n}{3} + \ldots + \binom{2n}{2n-1}$$