

INTRODUCTION TO DISCRETE MATHEMATICS. TEST #1

1. Verify if the following formulas are tautologies:  
 $(p \vee (q \wedge r)) \Rightarrow ((r \vee p) \wedge (p \vee q))$   
 $((p \Leftrightarrow q) \Leftrightarrow (p \Leftrightarrow p)) \Leftrightarrow (p \Leftrightarrow (q \Leftrightarrow p))$
2. Let  $X = \{21, 23, 25, \dots, 299\}$  be the set of odd integers lying between 20 and 300. Find the number of elements of the following set:  
 $Y = \{n \in X : ((3|n) \Rightarrow (n = 165)) \vee ((11|n) \Rightarrow (n = 165))\}$
3. Find the union  $\bigcup_{n \in \mathbb{N} \cup \{0\}} A_n$  and intersection  $\bigcap_{n \in \mathbb{N} \cup \{0\}} A_n$  of the sets

$$A_n = \left\langle 2 - \frac{1}{n^2 - 3}; 5 - \frac{1}{n + 2} \right\rangle.$$

4. Prove that the relation  $R$  defined on the set  $X = \mathbb{Z}$  by

$$mRn \equiv (2m)^2 + (3n)^2 \text{ is divisible by } 13$$

is an equivalence relation. Determine its equivalence classes  $[0]_R, [9]_R$ . How many different equivalence classes does this relation have?