1. Prove that the function $f : \mathbb{Z} \to \mathbb{N}$ defined as

$$f(n) = \begin{cases} n^2 - n + 1 & \text{for} \quad n \leqslant 0, \\ n^2 - n & \text{for} \quad n > 0 \end{cases}$$

is injective but not surjective.

2. Prove that the function $g : \mathbb{Z} \to \mathbb{N}$ defined as

$$g(n) = \begin{cases} n^2 + n + 1 & \text{for} \quad n \leqslant 0, \\ n^2 + n & \text{for} \quad n > 0 \end{cases}$$

is neither injective nor surjective.

3. Let $X = 3\mathbb{Z}$ be the set of all integers divisible by 3. Find bijections $f : \mathbb{Z} \to X$ and $g : X \to \mathbb{N}$ (and explain why they are bijections).

   (question for extra points) Find a bijection $h : (X \cup \mathbb{N}) \to (\mathbb{Z} \setminus X)$.

4. The set $G = \{0, 4, 6, 8\}$ is equipped with the operation $x \circ y$ defined as the last decimal digit of the number $48 + 2x + 2y - xy$.

   i) Compute $(4 \circ 4) \circ (4 \circ 4)$ ANSWER: 6

   ii) Find the neutral element of $\circ$ ANSWER: 6

   iii) Find the inverse element 4^{-1} ANSWER: 0

   iv) Is $(G, \circ)$ a group? ANSWER: yes

5. Let X be the set of all integers between 200 and 886 (including 200 and 886 themselves) that do not contain the digit 9.

   i) How many elements does X have? ANSWER: 565

   ii) How many elements of X have three even digits? ANSWER: 99

   iii) How many elements of X have three different digits? ANSWER: 392