

INTRODUCTION TO DISCRETE MATHEMATICS. TEST #2

1. Prove that the relation R defined on the set $X = \mathbb{Z}$ by

$$mRn \equiv (2m)^2 + (3n)^2 \text{ is divisible by } 13$$

is an equivalence relation. Determine its equivalence classes $[0]_R, [9]_R$. How many different equivalence classes does this relation have?

2. Let $X = 5\mathbb{Z}$ be the set of all integers divisible by 5, let $Y = \mathbb{Z}_+$ be the set of all positive integers. Find bijections $f : \mathbb{Z} \rightarrow X$ and $g : \mathbb{Z} \rightarrow Y$.

(question for extra points) Find a bijection $h : \mathbb{Z} \rightarrow (X \cup Y)$.

3. Determine for what values of the parameters $a, b \in \mathbb{R}$ the operation $x \dagger y = 2xy - 5x + ay + b$ defined for $x, y \in \mathbb{R}$ is commutative and associative.