Introduction to Discrete Mathematics Test #3, 25th Jan 2024

Name and surname .....

1. The set $G = \{0, 4, 6, 8\}$ is a group with the operation $x \circ y$ defined as the last decimal digit of the number $48 + 2x + 2y - xy$.

   i) Compute $(4 \circ 4) \circ (4 \circ 4)$ Answer: 6

   ii) Find the neutral element of $\circ$ Answer: 6

   iii) Find the inverse element 4^{-1} Answer: 0

2. Let X be the set of all integers between 200 and 886 (including 200 and 886 themselves) that do not contain the digit 9.

   i) How many elements does X have? Answer: 565

   ii) How many elements of X have three even digits? Answer: 99

   iii) How many elements of X have three different digits? Answer: 392

3. There are 12 blocks in a bag. They have the following letters on them: A A A C C C C D D T T T. We choose 4 blocks from the bag, and as we know there are $\binom{12}{4} = 495$ ways of doing that, ie. 495 four-element subsets. In how many of those 495 sets are there

   enough blocks to form the word CAT? Answer: 198

   enough blocks to form the word DAD? Answer: 24

   In this problem please write the actual numbers, eg. 24 rather than $\binom{4}{2}\binom{4}{3}$

4* (questions for extra points) Is the set G with the operation $\circ$ described in Problem 1 above a group? Answer: yes

INTRODUCTION TO DISCRETE MATHEMATICS TEST 3, JAN 25TH 2024

NAME AND SURNAME .....

1. The set $G = \{1, 3, 5, 7\}$ is a group with the operation $x \circ y$ defined as the last decimal digit of the number $9x + 9y - xy - 2$.

   i) Compute $(5 \circ 5) \circ (5 \circ 5)$ ANSWER: 3

   ii) Find the neutral element of $\circ$ ANSWER: 3

   iii) Find the inverse element 1^{-1} ANSWER: 7

2. Let X be the set of all integers between 300 and 996 (including 300 and 996 themselves) that do not contain the digit 8.

   i) How many elements does X have? ANSWER: 484

   ii) How many elements of X have three odd digits? ANSWER: 98

   iii) How many elements of X have three different digits? ANSWER: 336

3. There are 12 blocks in a bag. They have the following letters on them: A A A D D G G G O O O O. We choose 4 blocks from the bag, and as we know there are $\binom{12}{4} = 495$ ways of doing that, ie. 495 four-element subsets. In how many of those 495 sets are there

   enough blocks to form the word DAD?ANSWER: 24

   enough blocks to form the word DOG?ANSWER: 144

   In this problem please write the actual numbers, eg. 24 rather than $\binom{4}{2}\binom{4}{3}$

4* (questions for extra points) Is the set G with the operation $\circ$ described in Problem 1 above a group? ANSWER: yes