INTRODUCTION TO DISCRETE MATHEMATICS. ADDITIONAL HOMEWORK

- 1. For $G=\{3,5,7,9\}$ and $x\circ y$ defined as the last decimal digit of the number xy-x-y+2
 - i) show that \circ is a well-defined operation on G and present its table;
 - *ii*) find the neutral element of \circ ;
 - *iii*) prove that (G, \circ) is a group.
- 2. Let X be the set of all different ten-letter words that can be obtained by permuting the letters of the word BOOMERRANG (the misspelling is deliberate). How many elements does X contain? In how many of them there are no neighbouring identical letters?
- 3. How many six-digit natural numbers exist that contain only digits equal to one of the first three digits (eg. **375**553 and **707**007 are OK, while **123**182 is not, because it contains an 8, which does not appear among the first three digits).
- 4. In how many ways can we put 9 identical gold coins into four coloured boxes so that at least 1 goes into the blue box, at most 2 into red and at least 3 into green? The fourth box is yellow and it can contain any amount of coins (possibly 0).