Introduction to Discrete Mathematics (EIDMA)

sample combinatorial problems

1. Let X be the set of all different eight-letter words that can be obtained by permuting the letters of the word DEBUNKED.
   How many elements does X contain? $\frac{8!}{2!2!}$
   In how many of them there are neighbouring identical letters? $\frac{7!}{2!} + \frac{7!}{2!} - 6!$
   In how many elements of X there are no neighbouring vowels (terminology: E,U are vowel letters, while B,D,K,N are consonant letters)? $\binom{6}{3}\frac{5!}{2!}\frac{3!}{2!}$

2. Assume that a basketball line-up must consist of 5 players: 2 guards, 1 center and 2 forwards. A coach has in total 12 players in the team: there are 5 guards, 2 centers, 4 forwards, and additionally Peter Williams, who can play as center or forward. How many 5-player subsets of this 12-element set are acceptable line-ups? $\binom{5}{2}\binom{2}{1}\binom{4}{2} + \binom{5}{2}\binom{2}{0}\binom{4}{2} + \binom{5}{2}\binom{2}{1}\binom{4}{1}$
   In how many of them does Peter Williams appear? $\binom{5}{2}\binom{2}{0}\binom{4}{2} + \binom{5}{2}\binom{2}{1}\binom{4}{1}$

3. In how many ways can we put 20 identical silver coins into five coloured boxes so that at most 3 coins go into the blue box, at least 4 into red and at most 5 into green? The remaining boxes are yellow and black and may contain any number of coins. Every box except the red one may also remain empty. $\binom{20}{4} - \binom{16}{4} - \binom{14}{4} + \binom{10}{4}$

4. How many solutions of the equation $a+b+c+d = 80$ in integers greater than 0 satisfy simultaneously all the following conditions:
   $a \leqslant 30, 10 \leqslant c \leqslant 40$ and a, b, c, d are all even? $\binom{35}{3} - \binom{21}{3} - \binom{19}{3} + \binom{5}{3}$