Szereg Fouriera

Niech f będzie funkcją okresową: $f(x+2l)=f(x)$. Wtedy

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}),$$

gdzie

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}dx,$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}dx.$$

Wzór Greena

$$\int_C Pdx + Qdy = \int\int_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dxdy.$$

Warunki Cauchy'ego-Riemanna

Niech $z = x + jy, f(z) = u(x,y) + jv(x,y)$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \ \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$f'(z) = \frac{\partial u}{\partial x} + j\frac{\partial v}{\partial x}$$

Wzory całkowe Cauchy'ego

$$\int_{|z-z_0|=\epsilon} \frac{f(z)}{z-z_0}dz = 2\pi j f(z_0)$$

$$\int_{|z-z_0|=\epsilon} \frac{f(z)}{(z-z_0)^{n+1}}dz = 2\pi j\frac{f^{(n)}(z_0)}{n!}$$

Szeregi Taylora

$$f(z) = \sum_{n=0}^{\infty}\frac{f^{(n)}(z_0)}{n!}(z-z_0)^n$$

$$e^z = \sum_{n=0}^{\infty}\frac{z^n}{n!}$$

$$\cos z = \sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!}, \ \sin z = \sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!}$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n, \;\; \ln(1-z) = -\sum_{n=1}^{\infty} \frac{z^n}{n}$$

Transformata (przekształcenie całkowe) Laplace'a $\mathcal{L}$

$$\mathcal{L}[f(t)](s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$\mathcal{L}[f'(t)] = s\mathcal{L}[f(t)] - f(0)$$

$$\mathcal{L}[f''(t)] = s^2 L[f(t)] - sf(0) - f'(0)$$

$$\mathcal{L}[\int_0^t f(\tau) d\tau] = \frac{\mathcal{L}[f(t)]}{s}$$

$$\mathcal{L}[f(at)](s) = \frac{1}{a}\mathcal{L}[f(t)](\frac{s}{a})$$

$$\mathcal{L}[e^{at} f(t)](s) = \mathcal{L}[f(t)](s-a)$$

$$\mathcal{L}[t^n f(t)](s) = (-1)^n \frac{d^n \mathcal{L}[f(t)](s)}{ds^n}$$

$$\mathcal{L}[t^n e^{at}] = \frac{n!}{(s-a)^{n+1}}$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2}, \;\; \mathcal{L}[\sin at] = \frac{a}{s^2+a^2}$$

$$\mathcal{L}[f_1 * f_2] = L[f_1] L[f_2],$$

gdzie **splot** $*$ zadany jest dla $t > 0$ wzorem

$$(f_1 * f_2)(t) = \int_0^t f_1(\tau) f_2(t-\tau) d\tau.$$

Przydatne fakty

$$e^{x \pm jy} = e^x (\cos y \pm j \sin y)$$

$$\cos z = \frac{e^{jz} + e^{-jz}}{2}, \;\; \sin z = \frac{e^{jz} - e^{-jz}}{2j}$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos a \cos b = \frac{\cos(a+b) + \cos(a-b)}{2}$$

$$\sin a \sin b = \frac{\cos(a-b) - \cos(a+b)}{2}$$

$$\cos a \sin b = \frac{\sin(a+b) - \sin(a-b)}{2}$$