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# A Generic Metaheuristic Approach to Sequential Security Games

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# Problem definition

Sequential Security Games with Stackelberg Equilibrium

Defender (D) commits strategy first, then Attacker (A) chooses strategy

Goal: maximize Defender's payoff

$$BR(\pi^D) = \arg \max_{\pi^A \in \Pi^A} U^A(\pi^D, \pi^A)$$

$$\arg \max_{\pi^D \in \Pi^D} U^D(\pi^D, BR(\pi^D))$$

$\pi^D, \pi^A$  – Defender's/Attacker's strategy

$U^D, U^A$  – Defender's/Attacker's payoff

# Motivation and contribution

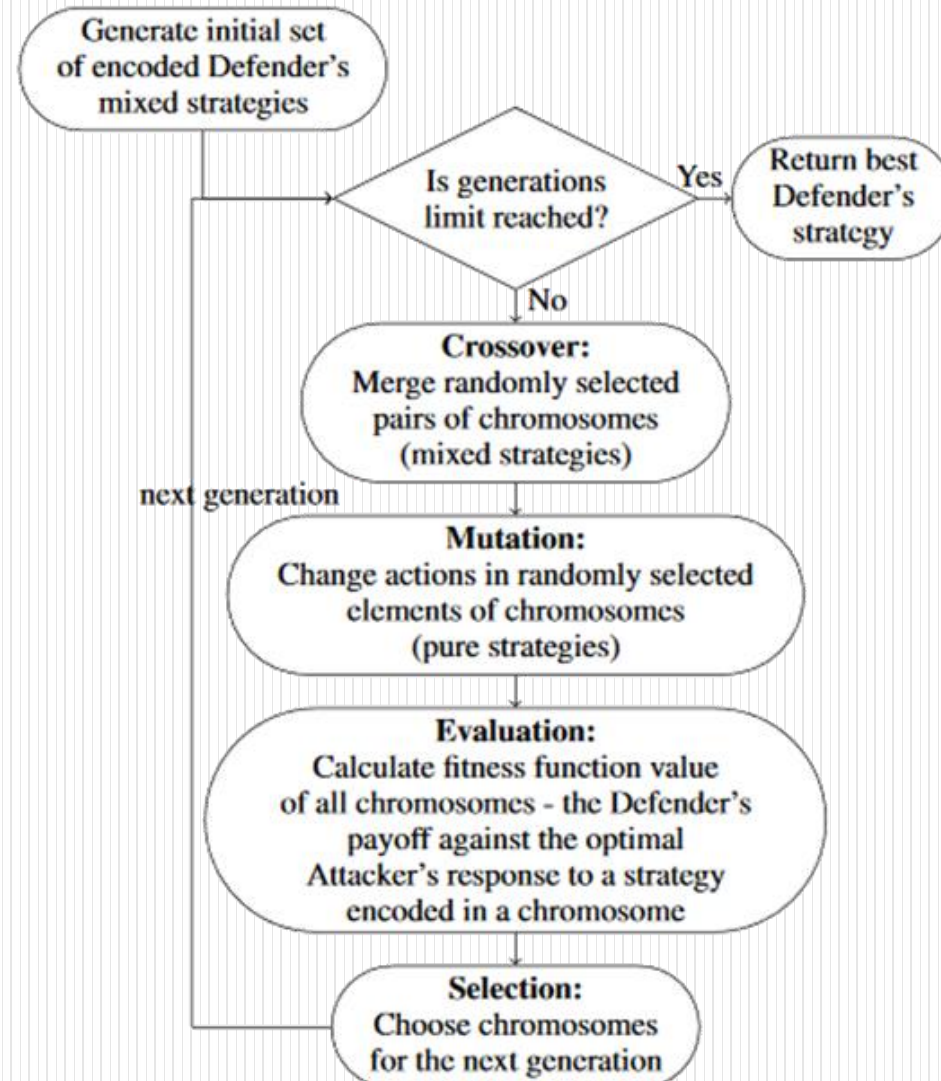
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Finding Stackelberg Equilibrium is a kind of optimization problem – **evolutionary algorithms** is one of the most promising optimization methods

Creating a **general** Stackelberg Games solution framework based on evolutionary algorithms, easily **adaptable to various types of games**

An **anytime** approximation method for **time-critical** applications

# EASG Overview



# Chromosome representation

Each chromosome represents Defender's mixed strategy – a set of pure strategies with their probabilities

Initially chromosomes contain random pure strategies

$$CH_q = \{(\pi_1^q, p_1^q), \dots, (\pi_{l_q}^q, p_{l_q}^q)\}, \sum_{i=1}^{l_q} p_{l_q}^q = 1$$

$\pi_i^q$  - pure strategy (e.g. list of Defender's actions in consecutive time steps)

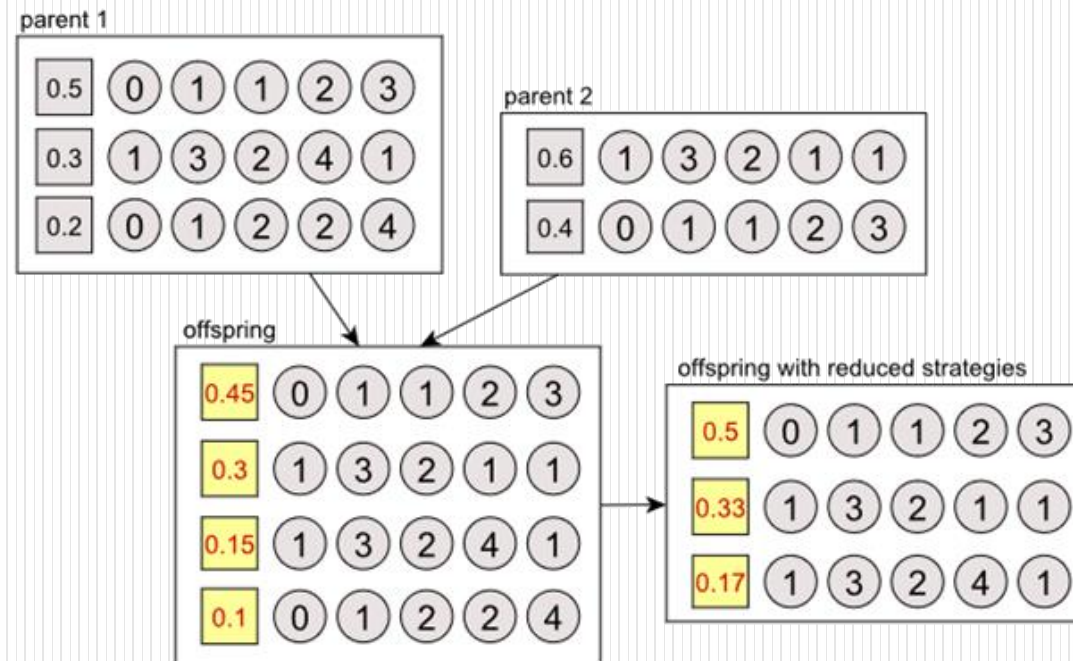
$p_i^q$  - probability of strategy  $\pi_i^q$

$l_q$  – length of chromosome  $CH_q$  (the number of pure strategies included in the mixed strategy represented by that chromosome)

# Crossover

Crossover operation combines two chromosomes by merging their sets of pure strategies and halving their probabilities

After crossover each pure strategy  $(\pi_j^q)$  may be deleted with probability  $1 - \frac{p_j^q}{2}$

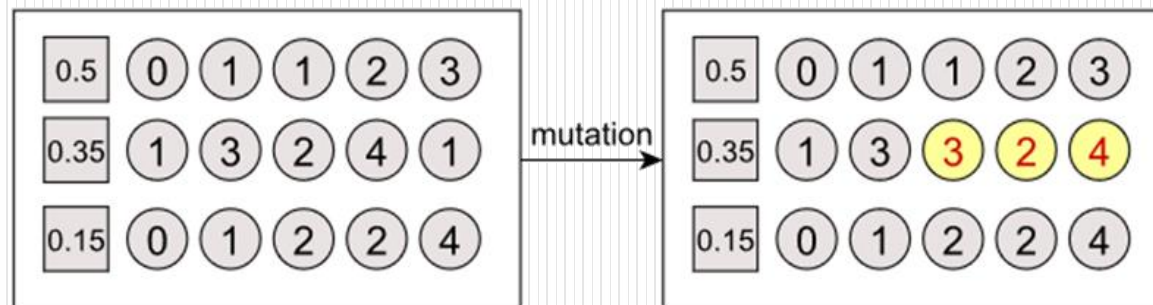


# Mutation

Mutation randomly changes one Defender's action in one of pure strategies starting from a randomly chosen time step

Each chromosome is mutated with *mutation rate* probability

Exploration of new areas of the search space



# Selection

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Fitness function - Defender's payoff in case of playing a mixed strategy encoded in the chromosome

Binary tournament - two chromosomes are randomly chosen and the one with a higher fitness value is promoted to the next generation with probability  $p_s > 0.5$ , otherwise the lower-fitted one is promoted

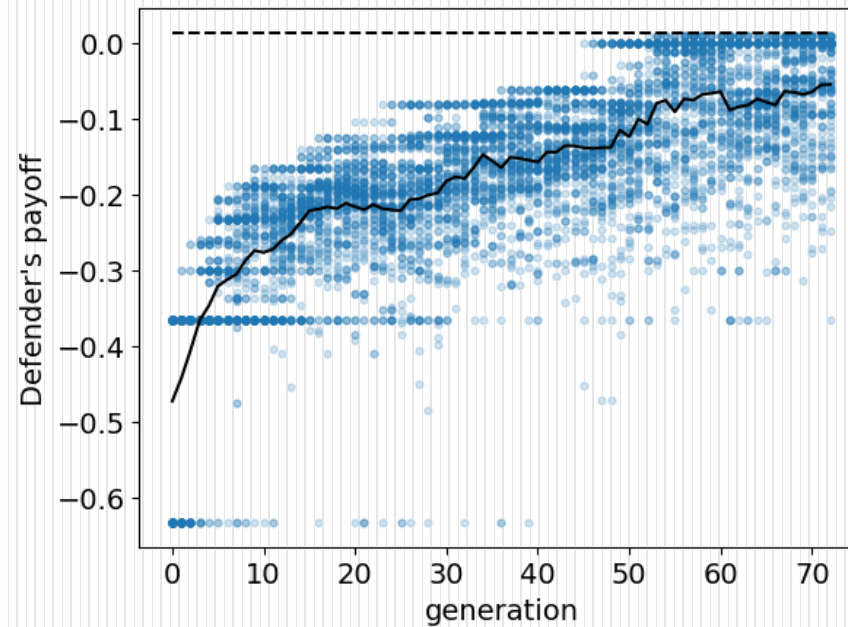
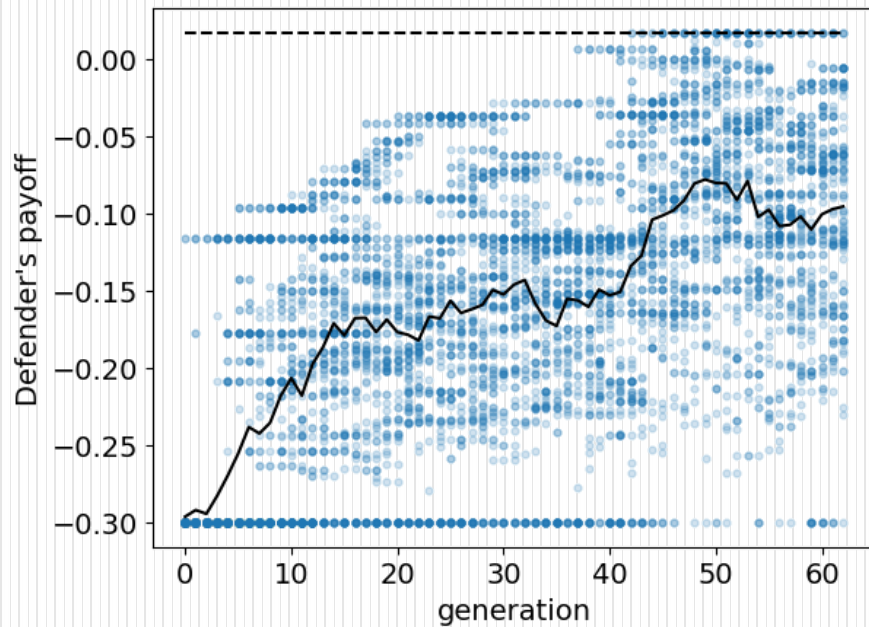
Some number of *elite* chromosomes (with the greatest fitness function value)  
→ unconditionally promoted to the next generation population



# Performance

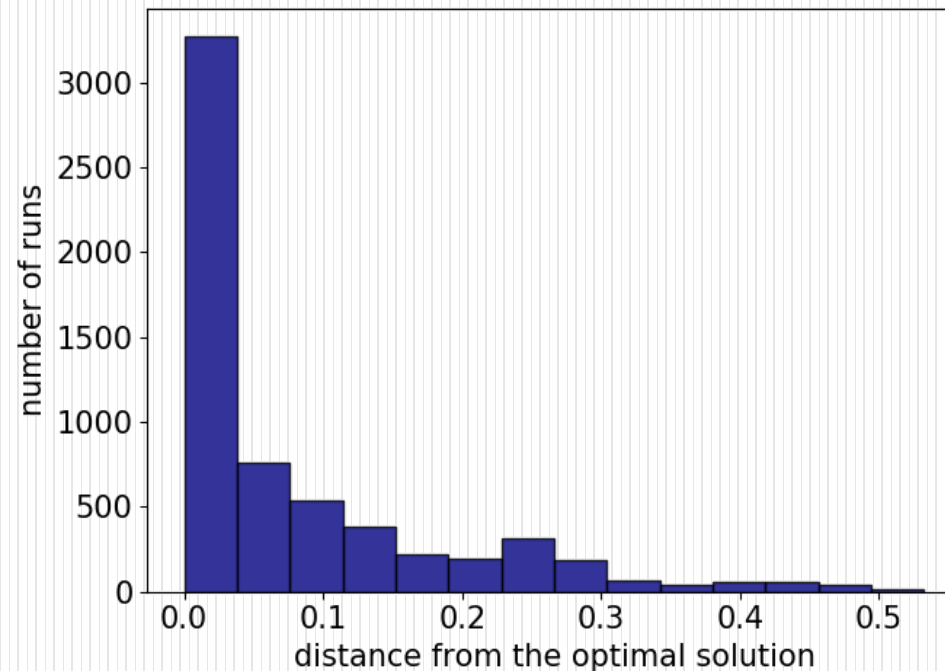
Mean Defender's payoff increases in time - the entire population moves towards the areas with higher payoff

Low-payoff individuals exist in all generations - exploration of new strategies



# Results quality

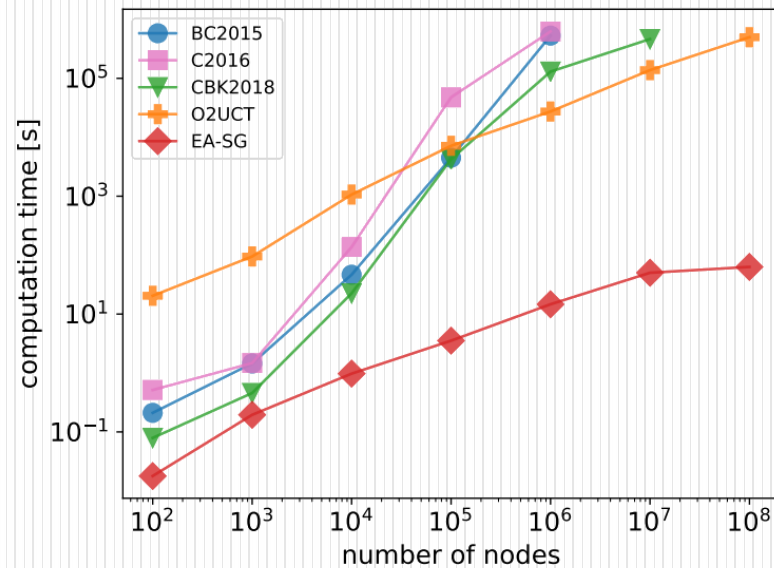
| Games type      | Fraction of games with optimal solution found | Mean difference between the optimal and EASG | The highest difference between the optimal and EASG |
|-----------------|---|--|---|
| Warehouse Games | 72%   | 0.0013                                       | 0.0127  |
| Search Games    | 47%   | 0.0253                                       | 0.0955  |
| Fliplt Games    | 73%   | 0.0087                                       | 0.0321  |



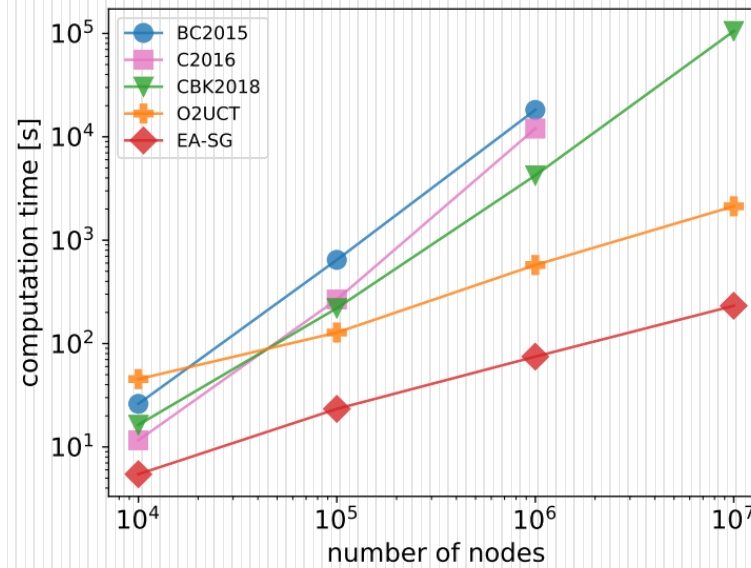
# Time scalability

Time performance strongly depends on selected steering parameters – the possibility of establishing the expected balance between computation time and quality of results

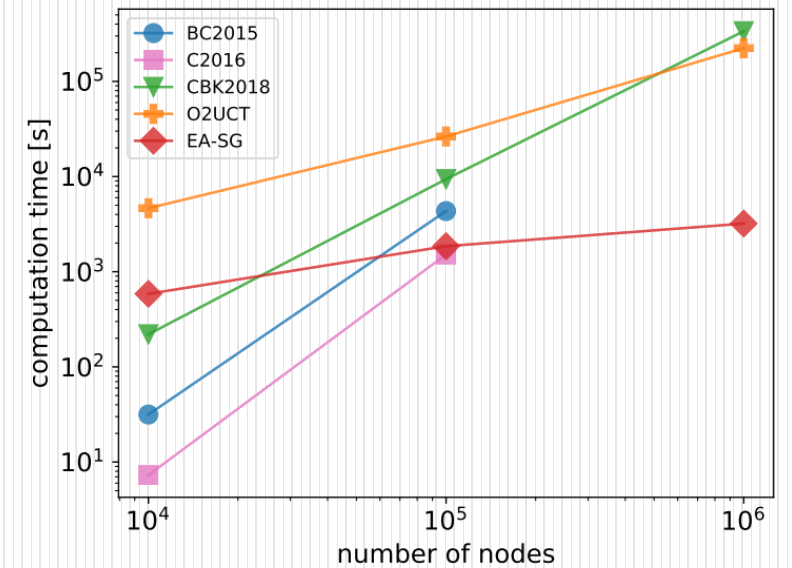
The highest time efficiency among the tested methods



Warehouse Games



Search Games



Flight Games

# Summary

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**Evolutionary algorithm** which can be **easily adapted to various types of Security Games**

Efficient approximation method with **high stability** and **good results quality**

Capable of **solving larger and more complex sequential Security Games** than state-of-the-art methods

Iteration-based construction - **well suited for time-critical applications**  
(*anytime* method)