

Diversity-driven Cooperating Portfolio of Metaheuristic Algorithms

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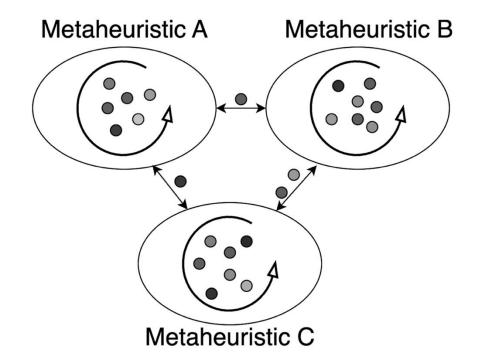
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Motivation SA DE Simulated Differential Annealing **PSO Evolution ACO** Particle Swarm **ABC** Optimization **Ant Colony** GA **Artificial Bee** Optimization Genetic Colony Algorithm and many more... Metaheuristic Algorithms Trendline. R² = 0.926 50 Number of Metaheuristic Algorithms taxonomy, applications, and open challenges"Artificial Intelligence Review 2023.

Idea

- Multiple metaheuristic into one framework to combine their strengths
- Inspiration from island genetic algorithms
- *n* islands each developed by a single metaheuristic



Limitations of Existing Approaches

 Traditional island models use a single metaheuristic (usually genetic algorithm) across all islands

Migration strategies often static and not diversity-aware

Lack of synergy and adaptive cooperation

Key questions in choosing migration strategies

When to migrate?

- How frequently should migration happen?
- What indicators trigger migration?

What to migrate?

- How many individuals should be migrated?
- Which individuals should be selected?
- How do different selection criteria affect algorithm performance?

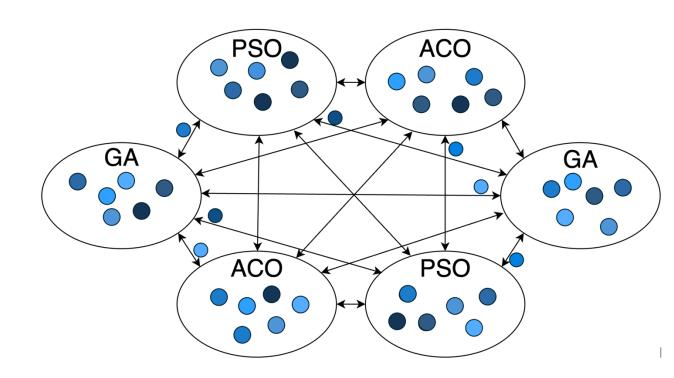
Where to migrate?

- How should the neighborhood between islands be defined?
- What connection topology should be chosen?

Diversity-driven Cooperating Portfolio of Metaheuristics (DdCPM)

Algorithm 1 Pseudocode of the baseline DdCPM method.

```
1: for each island I \in \mathcal{I} do
        initialize I_{population} with random individuals
 3: end for
 4: while evaluation\_budget > 0 do
        for each island I \in \mathcal{I} do
           if needs_migration(I) then
               I_{population} = I_{population} \cup migrate\_from(I_{neighbours})
           end if
 9:
        end for
10:
        for each island I \in \mathcal{I} do
11:
            I_{population} = \text{next\_generation}(I_{population}, I_{metaheuristic})
            evaluate(I_{population})
12:
            evaluation\_budget = evaluation\_budget - |I_{population}|
13:
14:
        end for
15: end while
16: return the best individual from I
```



When to migrate?

T1: Periodic migration every *x* iterations

T2: Average fitness doesn't increase for x iterations

T3: Maximum fitness doesn't increase for *x* iterations

T4: Average and maximum fitness don't increase for x iterations

T5: Diversity doesn't increase for x iterations

T6: Combination of T4 and T5

What to migrate?

S1: Individual with the best fitness

S2: Individual that increases diversity the most

S3: Random individual that increases diversity

S4: Combined sum of diversity increment and fitness

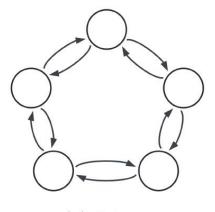
Where to migrate?

P1: Ring

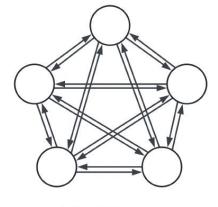
P2: Clique

P3: Cycle

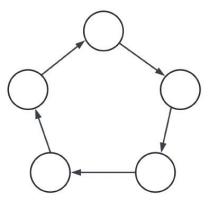
P4: Star



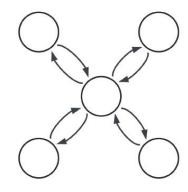
(a) Ring



(b) Clique



(c) Cycle



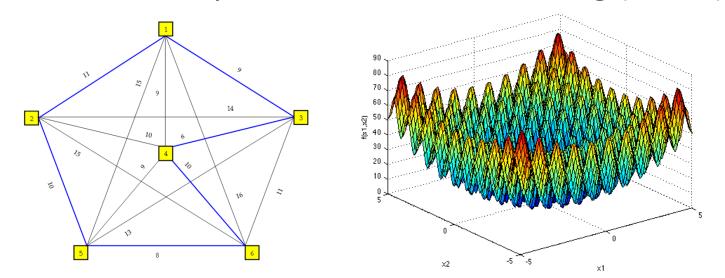
(d) Star

Experimental setup

- number of islands: 6 (2PSO, 2GA, 2ACO)
- population size: 100

Problems:

- discrete: Traveling Salesman Problem (TSP) 400-700 nodes
- continuous: Black-box Optimization Benchmarking (BBOB) 30 dimensions



Results

when to migrate

Periodic migration
Average fitness
Maximum fitness
Average and maximum fitness
Diversity
Combination of fitness and diversity

Γ	TSP	BBOB					
Avg rank	Avg fitness	Avg rank	Avg fitness				
15.9	35571	21.3	1.609				
49.2	35605	46.8	1.635				
11.6	35570	18.1	1.586				
18.0	35579	20.8	1.560				
29.4	35588	32.9	1.593				
7.2	35559	12.1	1.570				

what to migrate

Best fitness
Max diversity increasement
Random diversity increasement
Combined sum of fitness and diversity

Γ	TSP	ВВОВ					
Avg rank	Avg fitness	Avg rank	Avg fitness				
15.4	35580	41.3	1.609				
80.2	35623	59.9	1.672				
58.6	35610	54.8	1.646				
12.9	35571	25.3	1.584				

Results

where to migrate

Where	Т	TSP	ВВОВ			
variant	Avg rank	Avg fitness	Avg rank	Avg fitness		
Ring	12.9	35571	22.4	1.609		
Clique	26.4	35588	31.3	1.605		
Cycle	46.2	35605	49.3	1.649		
Star	40.9	35600	41.6	1.649		

Results - fitness-diversity balance

what to migrate

$$m = \alpha \cdot fitness + (1 - \alpha) \cdot diversity_increasement$$

α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
TSP	35558	35360	35335	35410	35397	35459	35594	35618	35680	35705	35742
BBOB	1.668	1.432	1.437	1.456	1.442	1.484	1.581	1.614	1.676	1.733	1.789

The impact of varying the balance parameter (α) between diversity and fitness in the selection of individuals for migration

Results – adaptive migration

- 1. Linearly decreasing α with iterations $\alpha = \frac{\text{current iteration}}{\text{total number of iterations}}$
- 2. Fitness stagnation-based adaptation

$$\alpha = \alpha + \delta$$
 if no improvement occurs for x_{δ} iterations

3. Fitness stagnation-based adaptation $\alpha = \frac{\max \text{ fitness}_{local}}{\max \text{ fitness}_{global}}$

TSP instance	rd400	fl417	pcb442	d493	att532	si535	u574	p654	d657	u724
$\alpha = 0.2$	15408 ± 18	11945 ± 12	51159 ±57	35234 ± 39	27845 ± 32	48894 ± 97	37244 ±65	34827 ± 38	49285 ± 49	42159 ±55
iterative decay	15319 ±27	11865 ±12	50813 ±68	35083 ±46	27739 ± 51	48665 ± 50	37077 ±52	34684 ±37	49138 ± 59	42047 ± 50
stagnation based	15359 ± 21	11908 ± 21	50872 ± 51	35000 ± 38	27746 ± 51	48738 ± 80	37146 ± 49	34724 ± 54	48991 ±50	41872 ± 74
fitness based	15368 ±19	11875 ± 18	50920 ± 75	35103 ± 40	27669 ±34	48600 ±90	37041 ±57	34612 ± 48	49132 ± 78	41948 ±60

BBOB instance	f2	f4	f6	f8	f10	f12	f14	f16	f18	f20	f22	f24
$\alpha = 0.1$	$0.014 \pm .000$	$1.392 \pm .010$	$0.904 \pm .005$	$1.857 \pm .011$	$3.183 \pm .045$	$4.459 \pm .045$	$0.633 \pm .005$	$0.801 \pm .006$	$0.747 \pm .007$	$1.428 \pm .015$	$0.439 \pm .005$	1.404 ±.009
iterative decay	$0.013 \pm .000$	$1.350 \pm .012$	$0.855 \pm .007$	1.728 ±.018	$3.053 \pm .020$	$4.359 \pm .057$	$0.608 \pm .004$	$0.771 \pm .011$	$0.702 \pm .007$	$1.377 \pm .017$	$0.428 \pm .004$	$1.313 \pm .014$
stagnation based	$0.013 \pm .000$	$1.349 \pm .017$	$0.870 \pm .009$	$1.789 \pm .020$	$3.113 \pm .022$	$4.322 \pm .063$	$0.599 \pm .004$	$0.773 \pm .008$	$0.706 \pm .008$	$1.340 \pm .011$	$0.425 \pm .003$	$1.316 \pm .008$
fitness based	$0.013 \pm .000$	1.303 ±.008	$0.885 \pm .010$	$1.788 \pm .024$	$3.022 \pm .032$	$4.350 \pm .038$	$0.609 \pm .008$	$0.755 \pm .005$	$0.722 \pm .005$	1.331 ±.019	$0.415 \pm .002$	$1.317 \pm .017$

Results

TSP instan	ice	rd400	fl417	pc	eb442	d493	att532	si535	u574	p65	64 de	657	u724
ACO	15	562 ±16	12094 ±11	514	55 ±97	35487 ±67	28102 ±49	49113 ±81	37477 ±	53 35141	±41 4957	'1 ±87 42	2495 ±47
GA	15	531 ±54	12095 ±33	5142	26 ±161	35441 ±125	28112 ± 84	49098 ±155	5 37360 ±1	41 35114	±121 4954	1 ±110 42	472 ±136
PSO	15	561 ±27	12108 ± 14	514	16 ±90	35518 ±58	28101 ± 27	49068 ±95	37412 ±	53 35164	±63 4956	60 ± 93 43	2436 ±62
ACO islan	ds 15	522 ±23	12070 ±25	5141	19 ±106	35489 ±46	28095 ± 47	49080 ± 71	37377 ±	54 35103	±69 4957	'8 ±86 42	2483 ±92
GA islands	s 15	480 ±48	12039 ±31	5133	32 ±126	35418 ±83	28040 ±61	49013 ±134	$4 37373 \pm$	94 35030	±80 4949	8 ±180 43	2380 ±91
PSO island	ls 15	527 ±17	12069 ±16	5139	91 ±106	35498 ±48	28064 ±48	49078 ±55	37431 ±	52 35130	±68 4953	31 ±57 42	506 ±107
ACO+GA+	-PSO 15	576 ±53	12112 ±31	5148	38 ±158	35497 ±122	28147 ±83	49123 ±153	3 37418 ±1	35162	±118 4959	2 ±109 42	485 ±135
DdCPM	15	448 ±23	11996 ±27	5129	94 ±107	35365 ±38	27987 ±29	48984 ±69	$37305 \pm$	87 35012	±37 494 1	18 ±62 42	2366 ±52
BBOB instance	f2	f4	f6		f8	f10	f12	f14	f16	f18	f20	f22	f24
ACO	0.015 ±.00	0 1.558 ±.	020 1.089 ±	:.010 2	2.228 ±.023	3.708 ±.067	$5.046 \pm .050$	0.707 ±.008	$0.859 \pm .015$	0.843 ±.010	1.613 ±.021	0.480 ±.006	5 1.492 ±.027
GA	$0.015 \pm .00$	0 1.491 ±.	053 1.050 ±	.035 2	$2.092 \pm .067$	$3.571 \pm .103$	$4.930 \pm .181$	$0.739 \pm .025$	$0.881 \pm .026$	$0.790 \pm .017$	$1.575 \pm .038$	$0.466 \pm .015$	$1.561 \pm .052$
PSO	$0.015 \pm .00$	0 1.579 ±.	024 1.060	:.018 2	$2.201 \pm .042$	$3.447 \pm .066$	$5.102 \pm .071$	$0.720 \pm .008$	$0.889 \pm .015$	$0.833 \pm .010$	$1.574 \pm .024$	$0.466 \pm .006$	1.511 ±.019
ACO islands	$0.015 \pm .00$	0 1.466 ±.	019 1.008	:.022	$2.132 \pm .040$	$3.466 \pm .038$	$4.706 \pm .088$	$0.699 \pm .013$	$0.872 \pm .018$	$0.817 \pm .010$	$1.573 \pm .029$	$0.487 \pm .008$	$1.574 \pm .028$
GA islands	0.014 ±.00	00 1.457 ±.	050 1.041	:.028 2	$2.058 \pm .061$	$3.546 \pm .124$	$4.993 \pm .122$	$0.683 \pm .018$	$0.859 \pm .029$	$0.781 \pm .026$	$1.514 \pm .047$	$0.468 \pm .018$	$3 1.488 \pm .039$
PSO islands	$0.015 \pm .00$	00 1.528 ±.	040 1.003	:.018 2	$2.017 \pm .050$	$3.669 \pm .071$	$5.007 \pm .117$	$0.690 \pm .010$	$0.887 \pm .015$	$0.817 \pm .012$	$1.509 \pm .019$	$0.481 \pm .011$	$1.492 \pm .016$
ACO+GA+PSO	$0.016 \pm .00$	00 1.495 ±.	051 1.053	:.034 2	$2.098 \pm .065$	$3.580 \pm .101$	$4.945 \pm .178$	$0.741 \pm .024$	$0.885 \pm .025$	$0.792 \pm .016$	$1.580 \pm .037$	$0.468 \pm .014$	$1.565 \pm .050$
DdCPM	$0.014 \pm .00$	00 1.411 ±.	023 0.927 :	±.011 1	1.886 ±.036	$3.229 \pm .057$	4.639 ±.108	$0.653 \pm .009$	$0.818 \pm .011$	$0.777 \pm .016$	$1.450 \pm .024$	$0.444 \pm .009$	9 1.421 ±.026

Comparison of hybridized metaheuristics (DdCPM) with individual methods for TSP (top) and BBOB (bottom) instances

Summary

- A novel hybrid framework combining metaheuristics via adaptive, diversity-aware migration
- Outperforms traditional and island-based models
- Diversity is essential for avoiding stagnation and improving solution quality
- Combination of fitness and diversity yields the best results
- Strong results across different problem types

THANK YOU



Full paper

