



Optimized mutation operator in evolutionary approach to Stackelberg Security Games

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Security Games

- Two asymmetrical players: **Defender and Attacker**
- Each game is composed of **m time steps**.
- Each player chooses an action to be performed in each time step.
- A player's pure strategy σ_p ($P \in \{D, A\}$) is a sequence of their actions in consecutive time steps: $\sigma_p = (a_1, a_2, \dots, a_m)$.
- **Defender commits to his/her strategy first.**
- Attacker, **knowing the Defender's strategy**, chooses his/her strategy.
- Defender always commits to a mixed strategy.

Stackelberg equilibrium

Stackelberg equilibrium: a pair of players' strategies, for which strategy change by any of players leads to his/her result deterioration.

$$(\pi_D^*, R(\pi_D^*)) \in \Pi_D \times \Pi_A$$

$$\pi_D^* = \operatorname{argmax}_{\pi_D \in \Pi_D} U_D(\pi_D, R(\pi_D))$$

$$R(\pi_D) = \operatorname{argmax}_{\pi_A \in \Pi_A} U_A(\pi_D, \pi_A)$$

$G \in \{D, A\}$ - players (Defender, Attacker)

Π_G - a set of player's G all mixed strategies

U_G - payoff of player G

Goal: find optimal Defender's strategy

Real-life applications



Federal Air Marshal Service



US Coast Guard in Boston Harbor



Los Angeles Airport

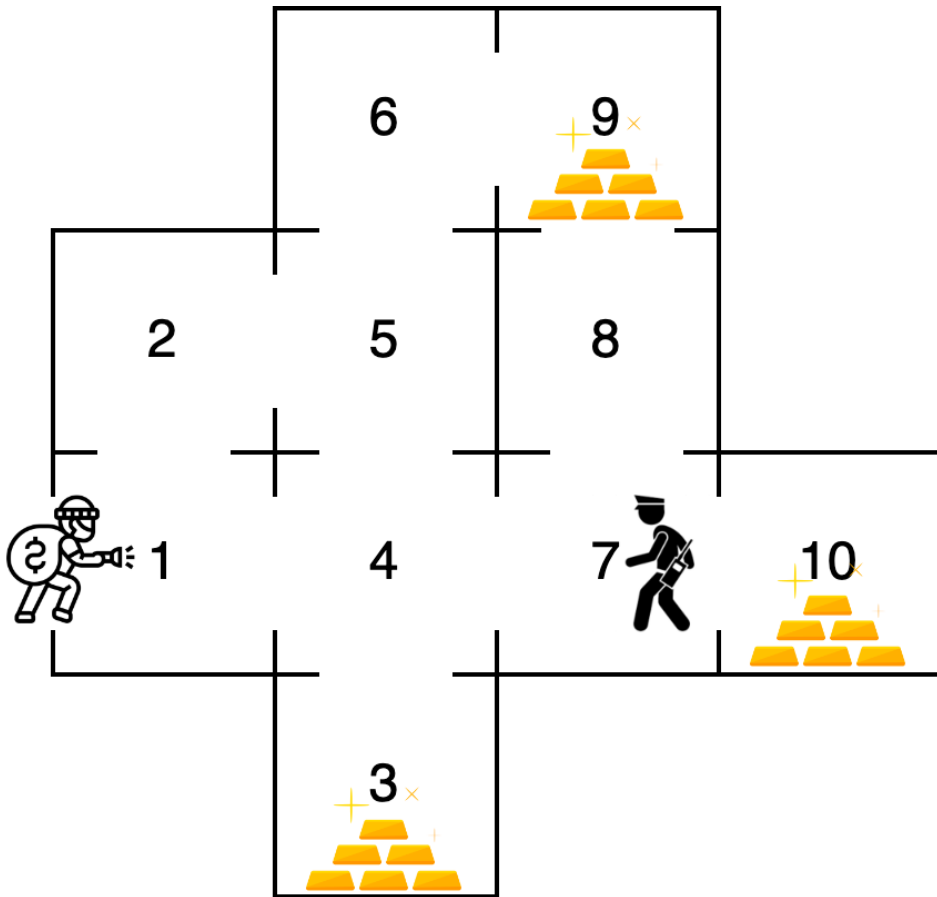


Poaching in Uganda

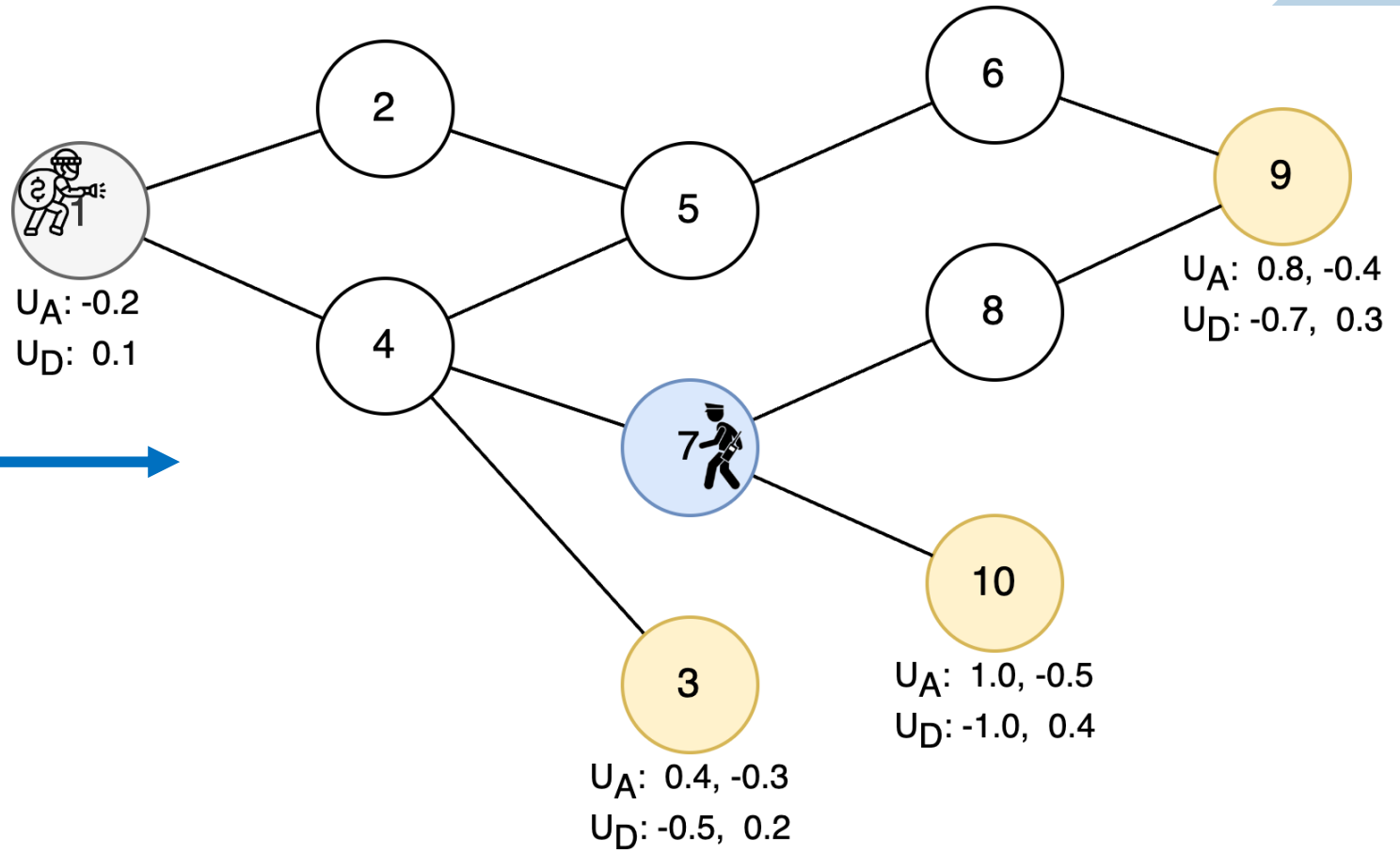
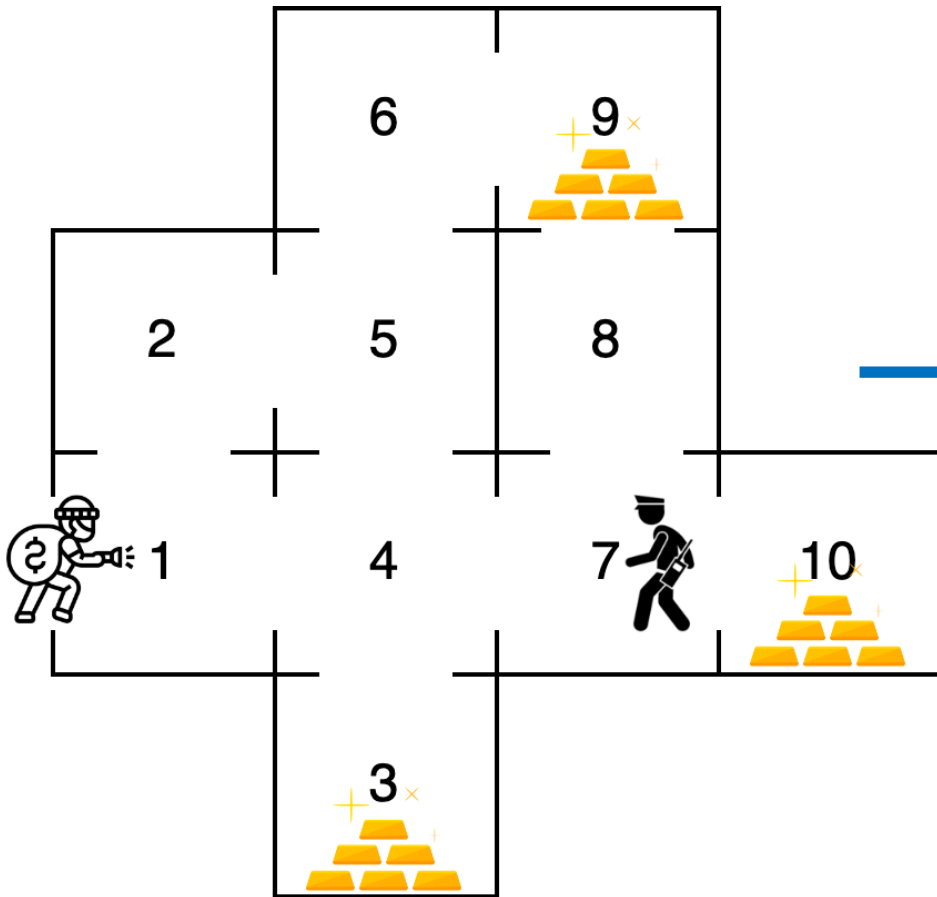


Tickets control in Los Angeles

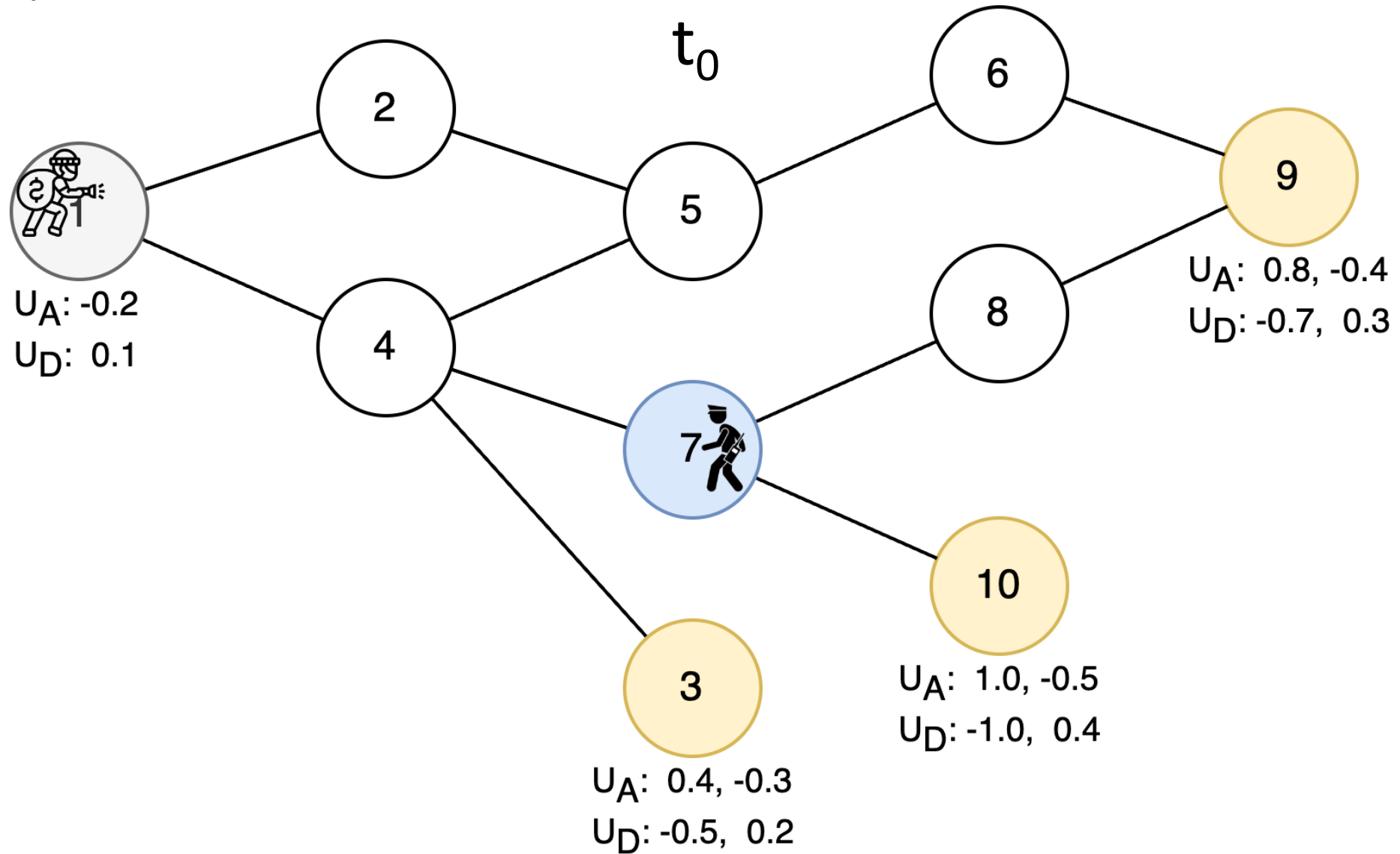
Example



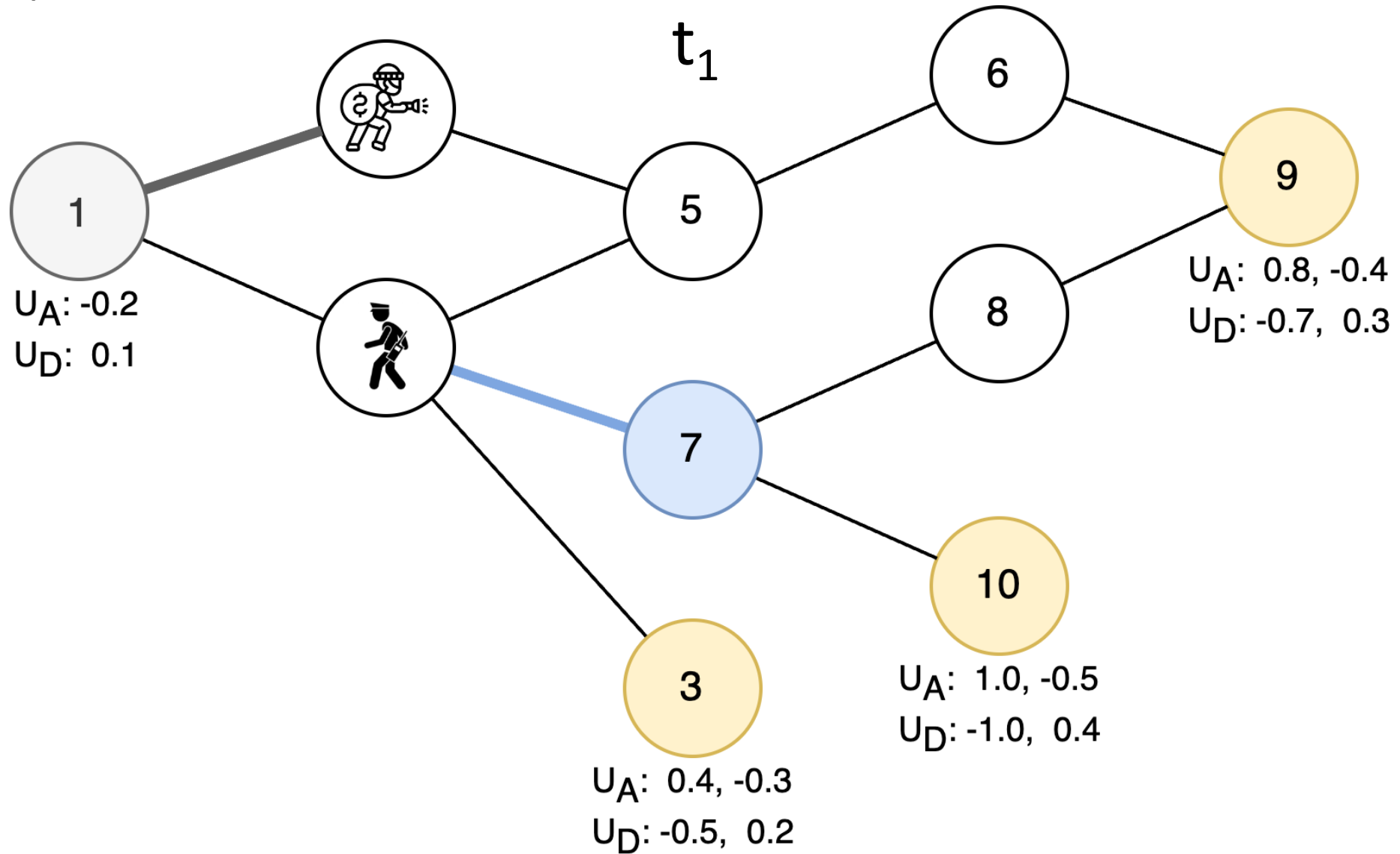
Example



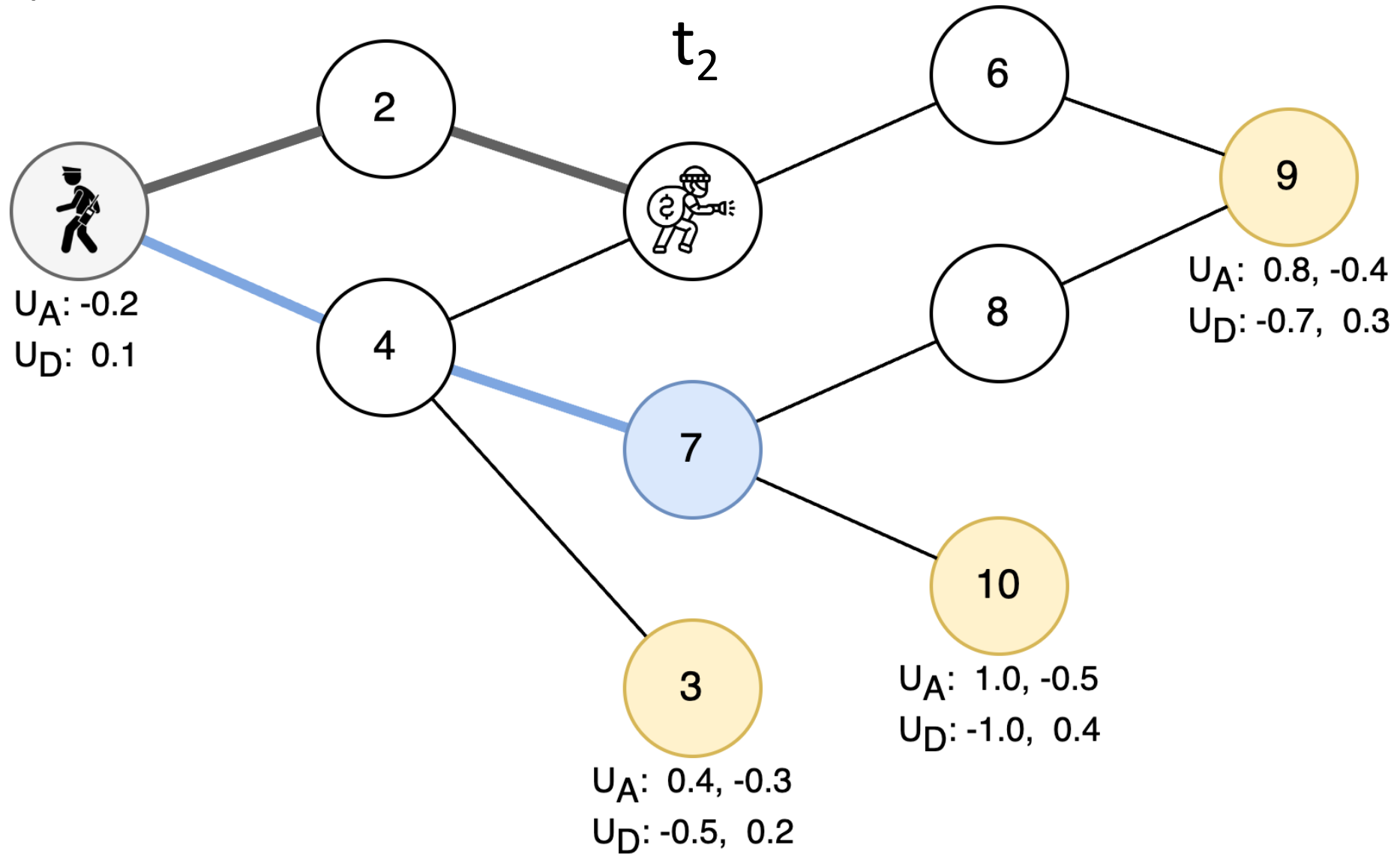
Example – scenario 1



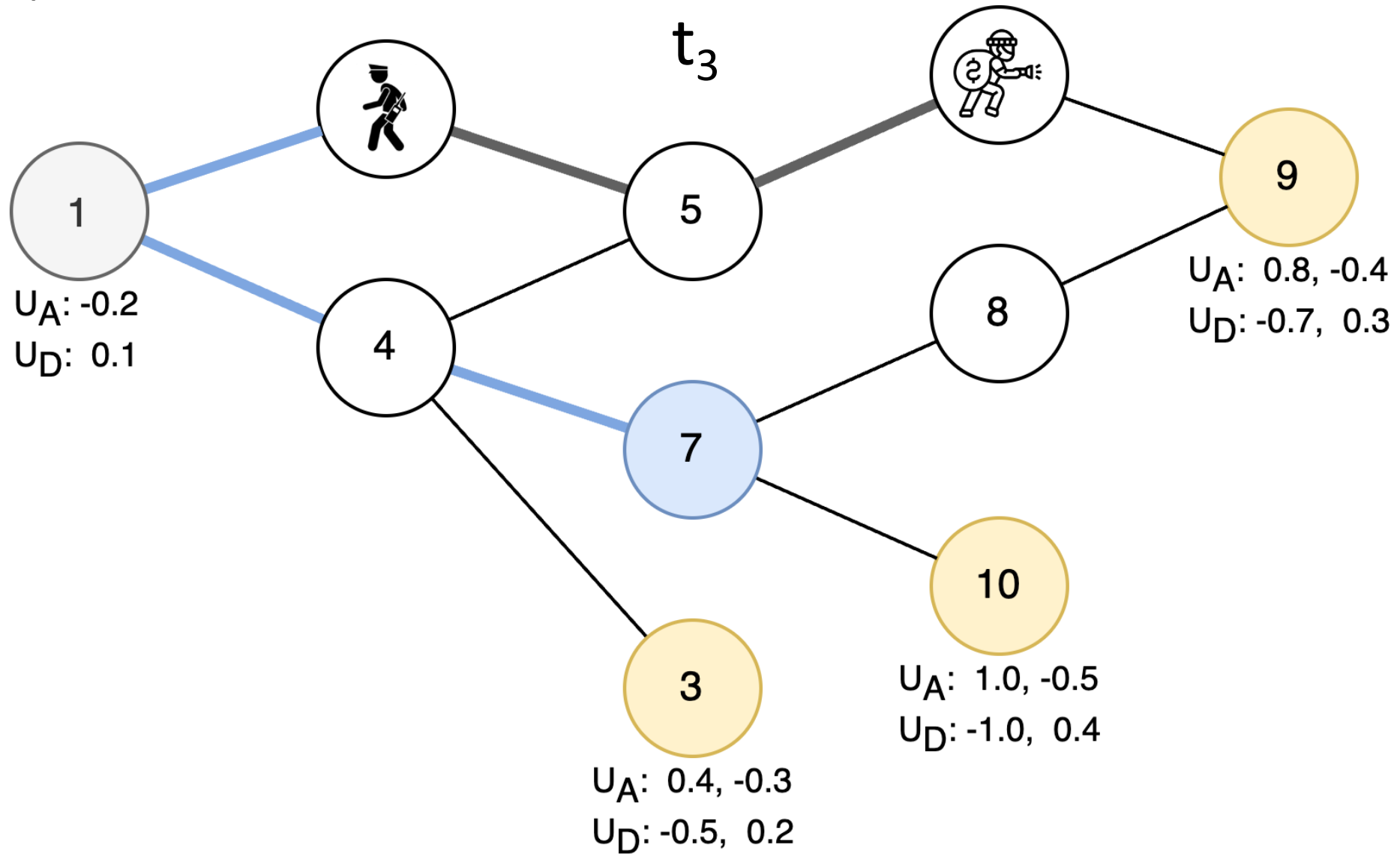
Example – scenario 1



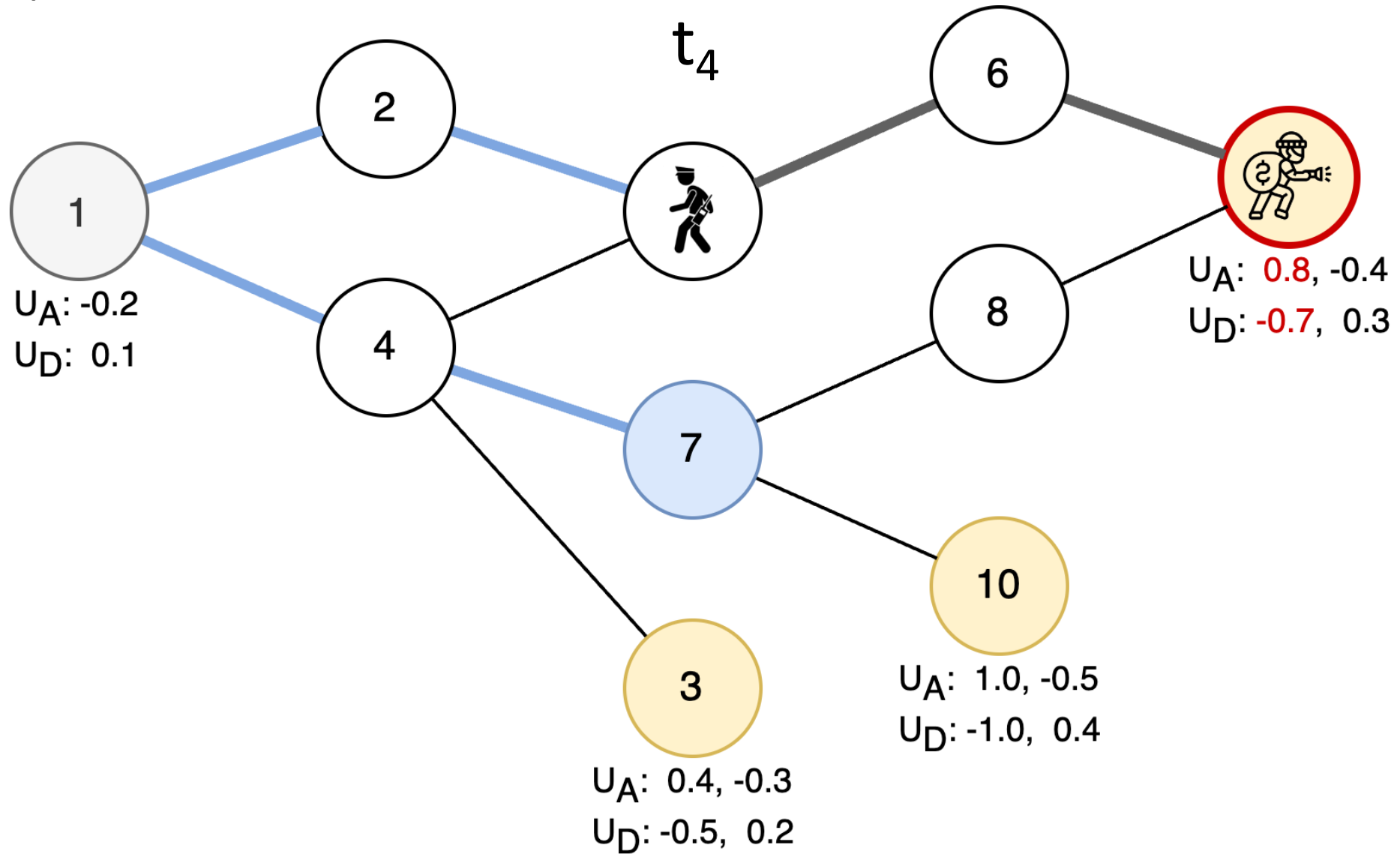
Example – scenario 1



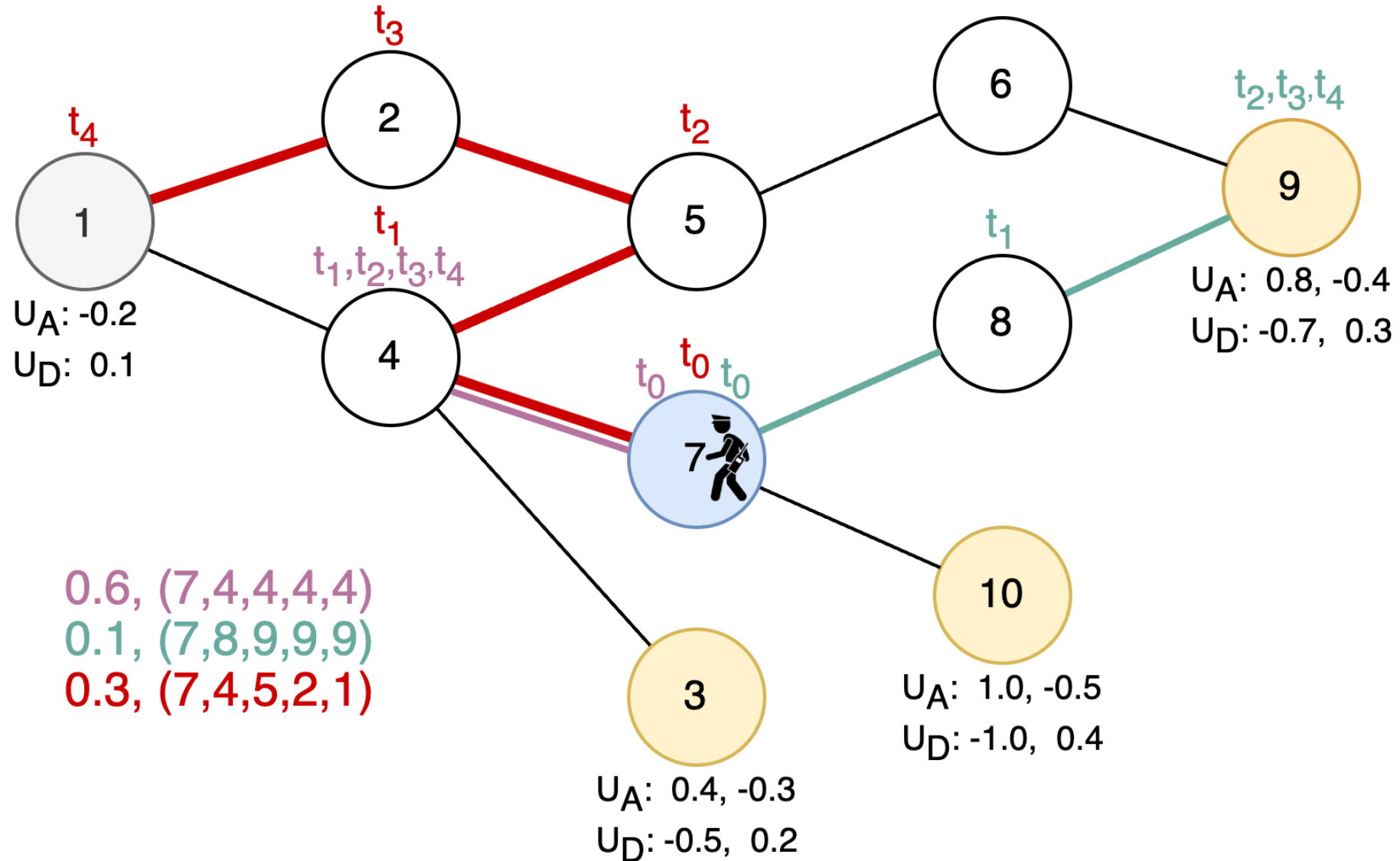
Example – scenario 1



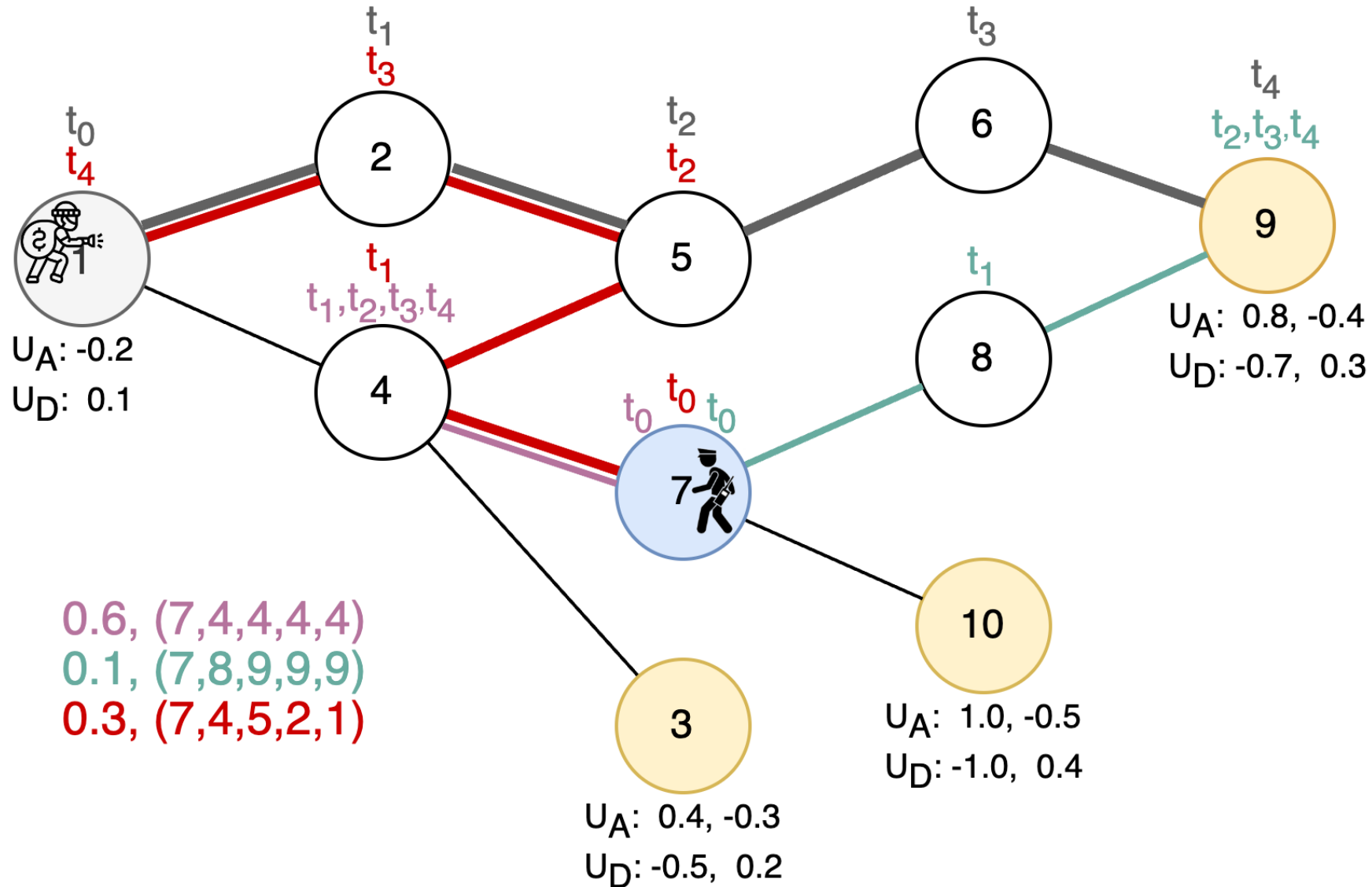
Example – scenario 1



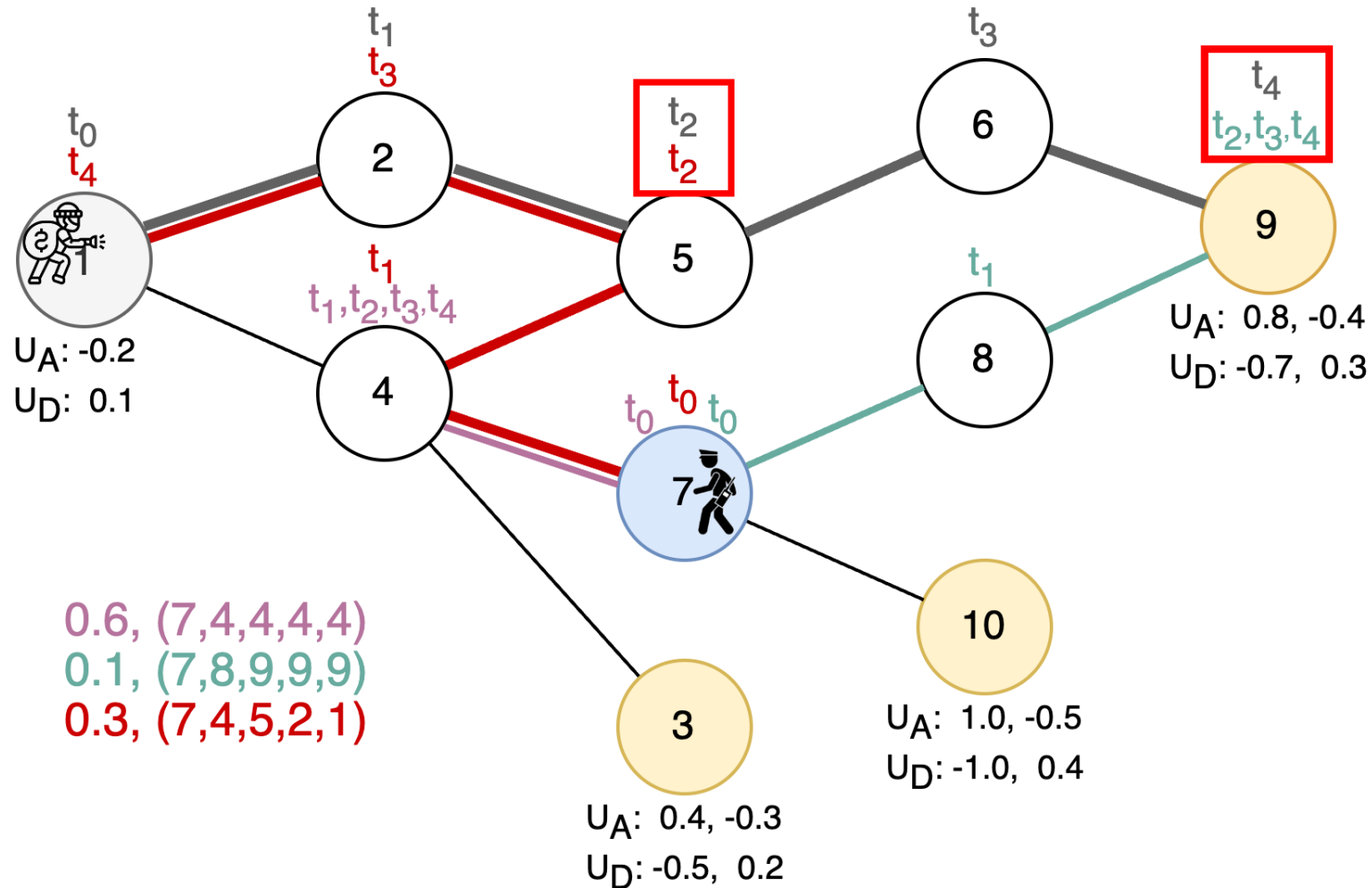
Example - mixed strategy



Example - mixed strategy



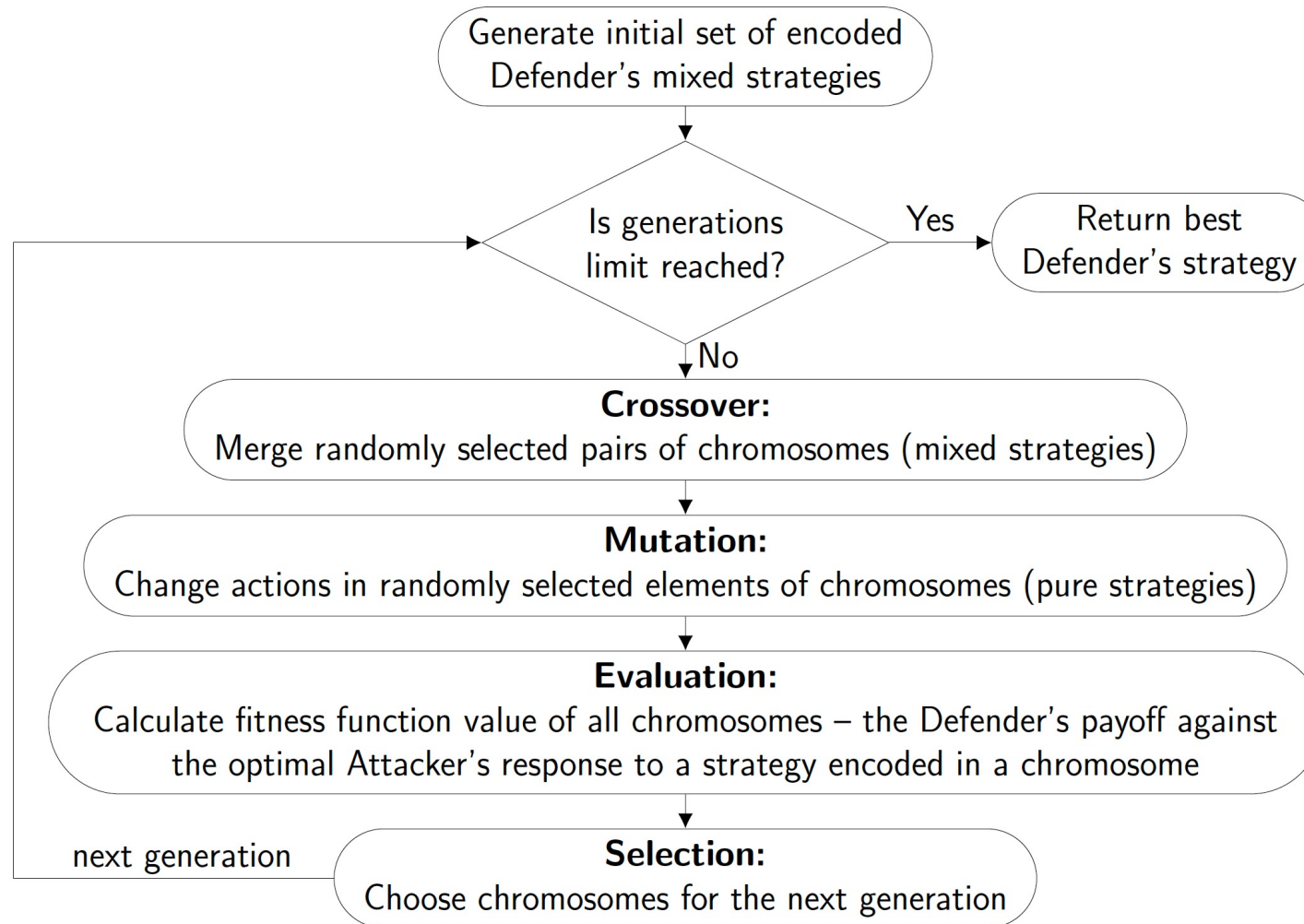
Example - mixed strategy



$$U_A(\pi_D, \pi_A) = 0.3 \cdot -0.2 + 0.1 \cdot -0.4 + 0.6 \cdot 0.8 = 0.38$$

$$U_D(\pi_D, \pi_A) = 0.3 \cdot 0.1 + 0.1 \cdot 0.3 + 0.6 \cdot -0.7 = -0.36$$

Evolutionary Algorithm for Security Games (EASG)



EASG - crossover

- Crossover role: combining existing solutions
- Each individual takes part in crossover with crossover rate probability p_k

$$CH_{1-2} = \left\{ \left(\sigma_1^1, \frac{p_1^1}{2} \right), \dots, \left(\sigma_{l_1}^1, \frac{p_{l_1}^1}{2} \right), \left(\sigma_1^2, \frac{p_1^2}{2} \right), \dots, \left(\sigma_{l_2}^2, \frac{p_{l_2}^2}{2} \right) \right\}$$

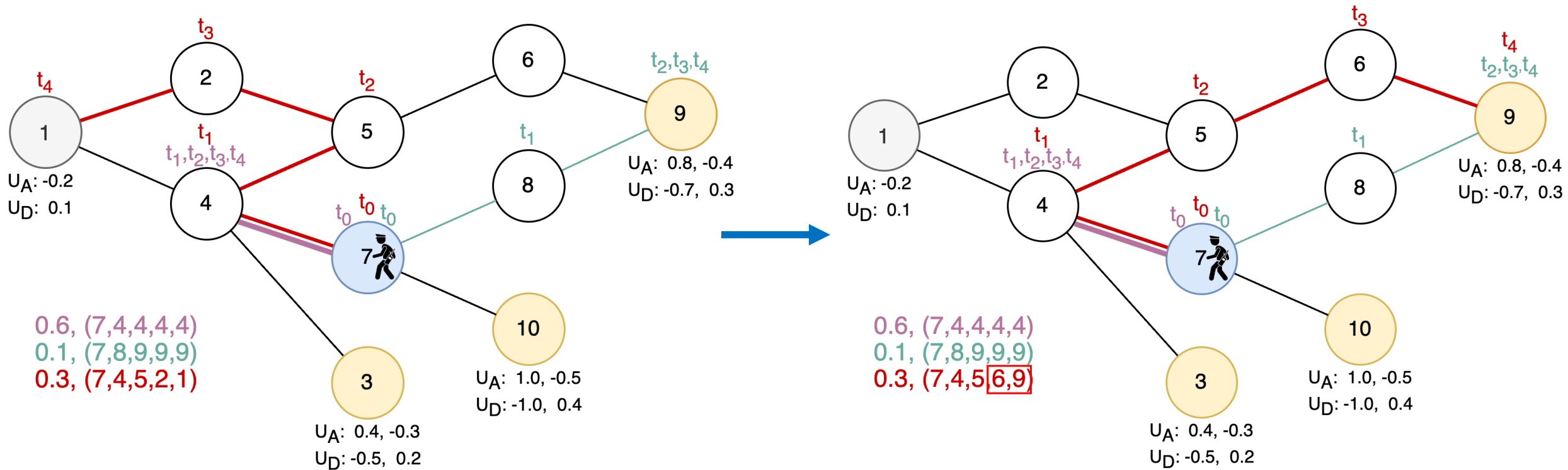
- After crossover each pure strategy may be deleted with probability equal to $(1 - p_i^q)^2$

EASG - mutation

- Mutation role: introduce some random perturbation to explore new areas of the search space
- Each individual is mutated with mutation rate probability p_m
- Random pure strategy σ_i^q is chosen which is modified starting from the random time step

$$\sigma_i'^q = (a_1, a_2, \dots, a_{s-1}, a'_s, a'_{s+1}, \dots, a'_m)$$

EASG mutation - example



Mutation enhancements

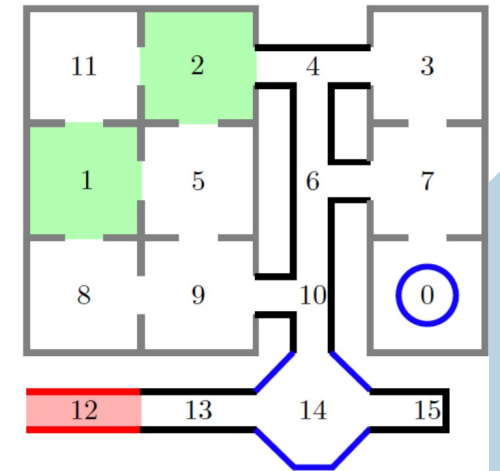
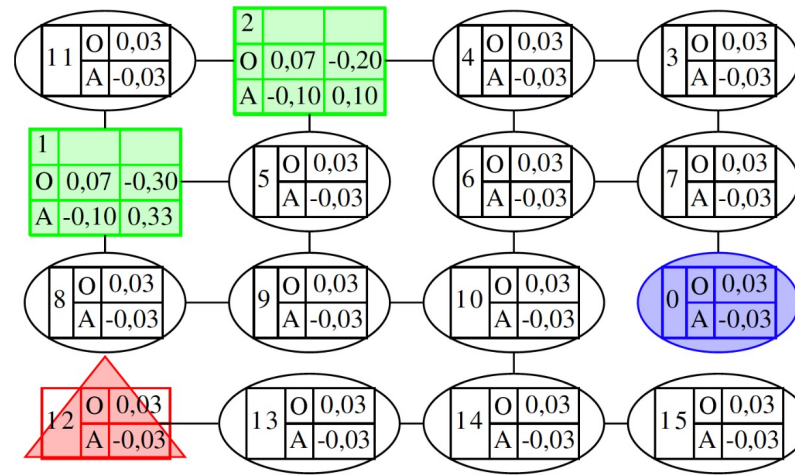
- **EASG_n** - EASG algorithm with repeated mutation.
- **MANPS₁, MANPS_n** - *mutation adds new pure strategy* - a uniformly selected pure strategy is added with a uniformly sampled probability.
- **MCP₁, MCP_n** - *mutation changes probability* - a probability of randomly selected pure strategy is uniformly changed.
- **MSP₁, MSP_n** - *mutation switches probability* - probabilities of two randomly chosen pure strategies are switched.
- **MDPS₁, MDPS_n** - *mutation deletes pure strategy* - a randomly chosen pure strategy is removed.
- **MCWPS** - *mutation changes the weakest pure strategy* - mutation is applied only to a pure strategy with the lowest payoff.
- **MDWPS** - *mutation deletes the weakest pure strategy* - pure strategy with the lowest payoff is deleted

Experimental setup

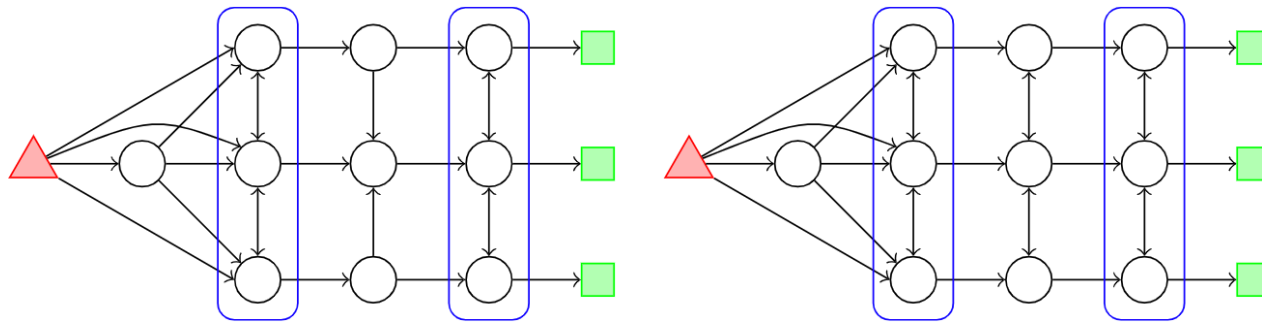
300 test game instances of 3 types:

- 150 Warehouse Games (WHG)
- 90 Search Games (SEG)
- 60 Flipt Games (FIG)

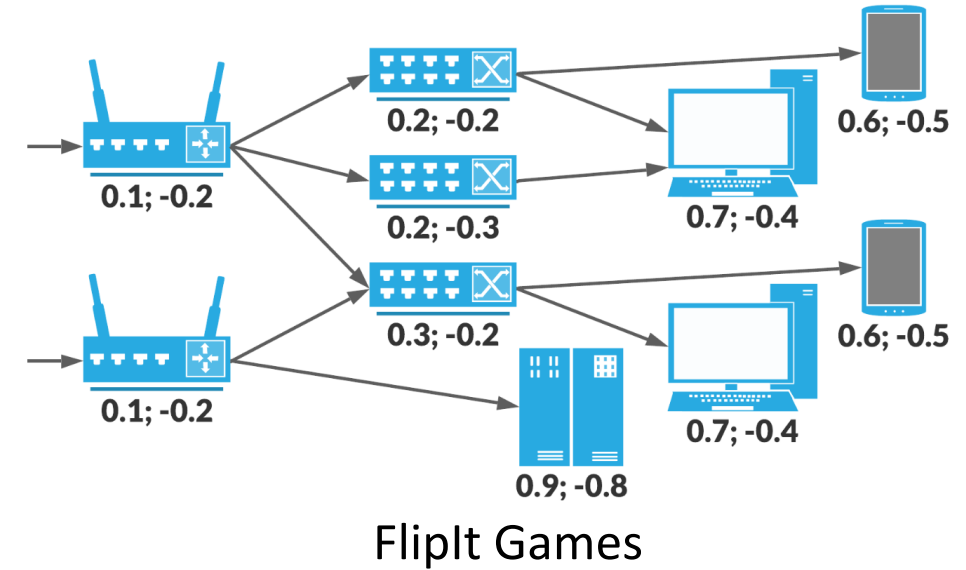
30 independent runs for each game instance



Warehouse Games



Search Games



Flipt Games

Results

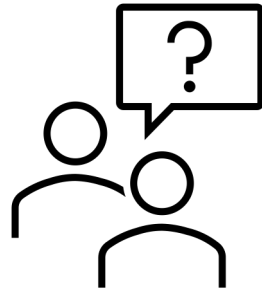
	Defender's payoff			Computation time [s]		
	WHG	SEG	FIG	WHG	SEG	FIG
EASG	0.017	0.108	0.031	152	2534	328
EASG _n	0.017	<u>0.135</u>	<u>0.037</u>	1206	21913	3051
MANPS ₁	0.014	0.059	0.031	156	2548	313
MANPS _n	0.016	<u>0.139</u>	<u>0.036</u>	1366	21892	2988
MCP ₁	0.015	0.074	0.030	148	2422	336
MCP _n	0.016	<u>0.131</u>	<u>0.037</u>	1285	22651	3008
MSP ₁	<u>0.013</u>	0.099	0.024	156	2583	316
MSP _n	0.016	0.108	<u>0.037</u>	1332	21447	2931
MDPS ₁	<u>0.013</u>	<u>0.052</u>	<u>0.029</u>	147	2620	313
MDPS _n	<u>0.013</u>	<u>0.053</u>	<u>0.026</u>	1283	22026	2900
MCWPS	<u>0.013</u>	<u>0.046</u>	0.030	148	2612	321
MDWPS	<u>0.008</u>	<u>0.058</u>	<u>0.018</u>	139	2361	299

The average Defender's payoff and the computation time for various mutation operators. The best results are **bolded**. Results that are better than the baseline version of the algorithm (EASG) are underlined. In cases where the difference between the baseline version (EASG) and a given variation is statistically significant the result is highlighted with a gray background.

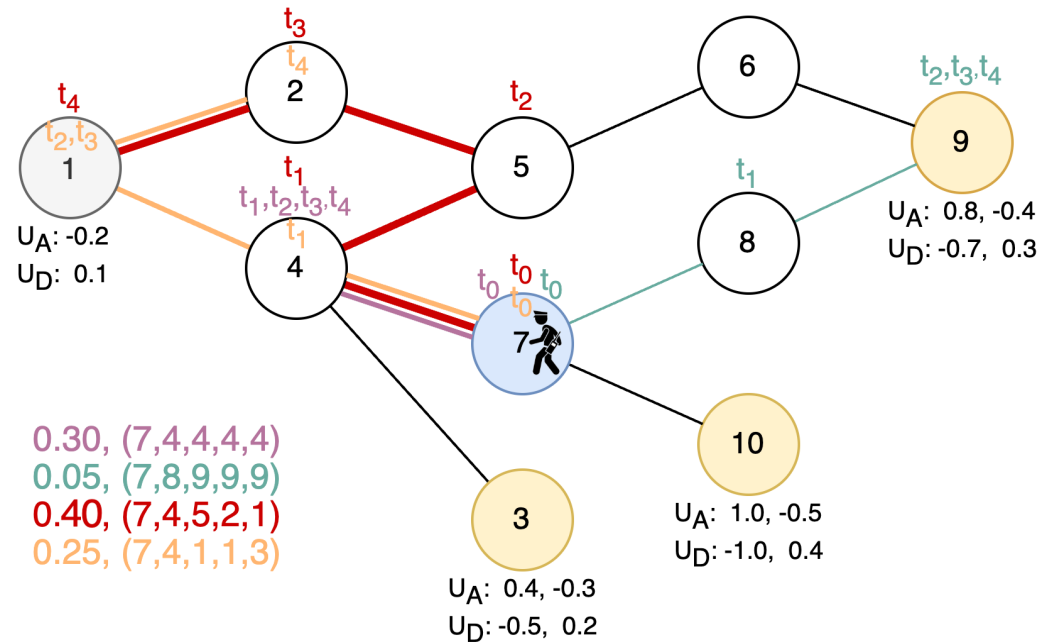
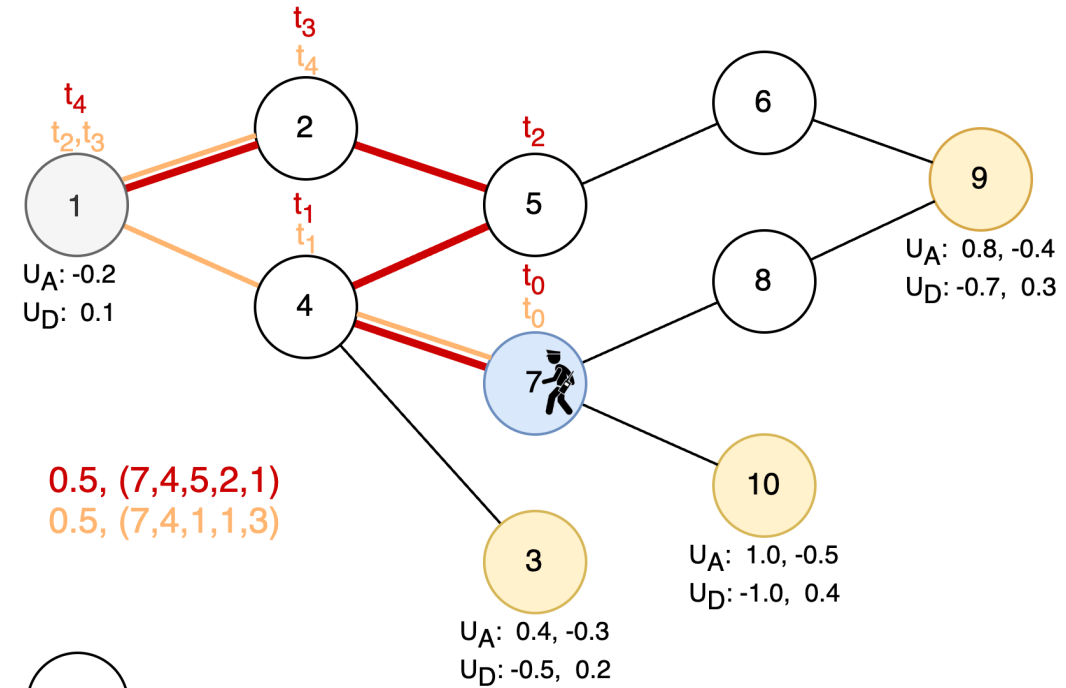
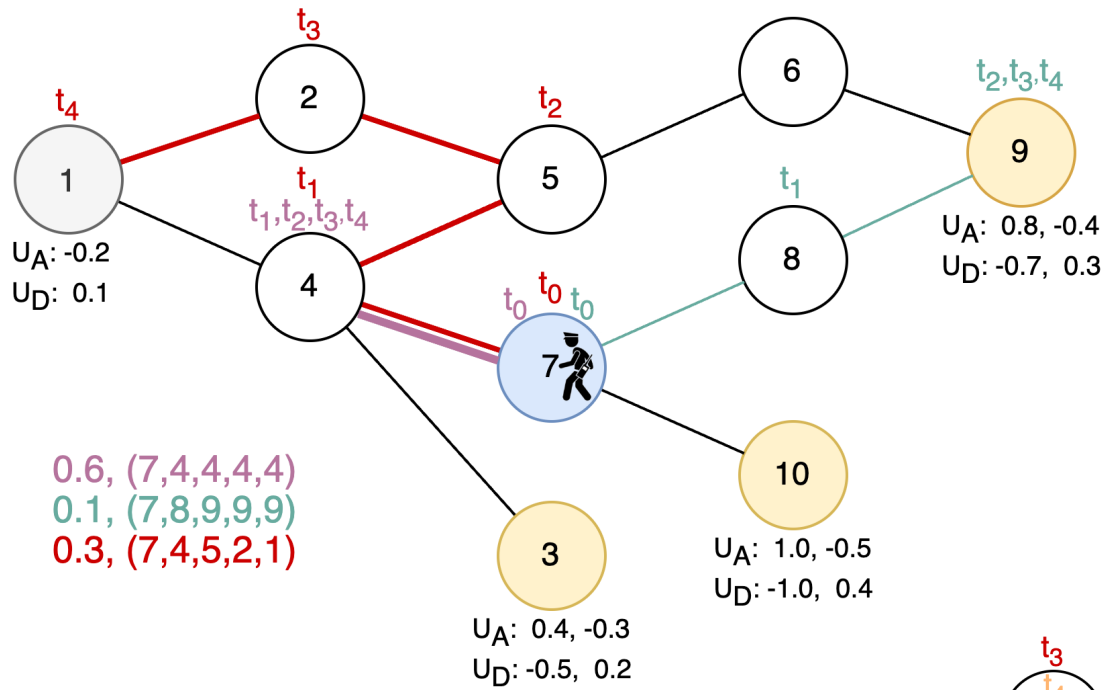
Conclusions

- **Repetition of mutation operation leads to improvement of SSGs outcomes**, though at the expense of significant increase in computation time.
- The proposed modifications offer a **viable alternative to the base EASG formulation** for cases when computational cost is less important.

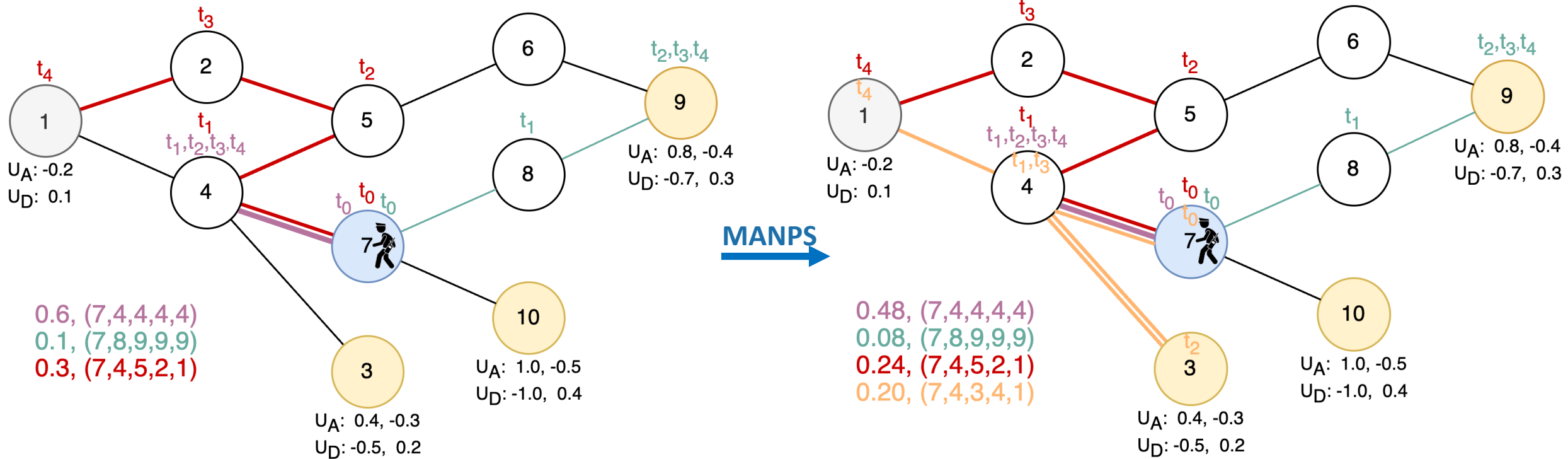
Thank you



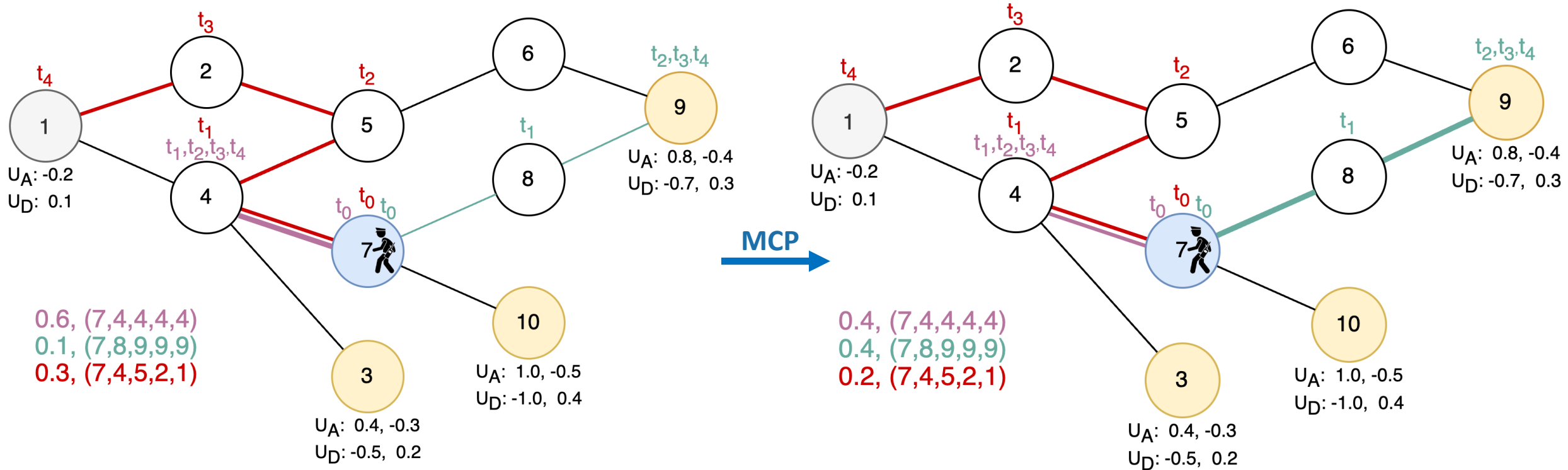
EASG crossover - example



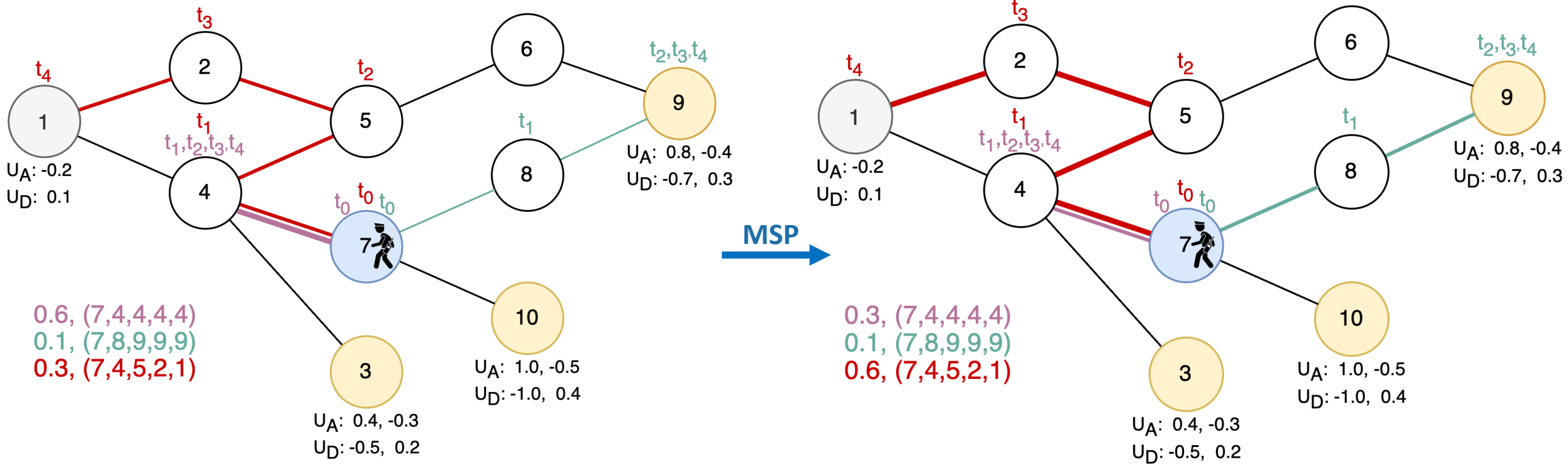
MANPS - mutation adds new pure strategy - a uniformly selected pure strategy is added with a uniformly sampled probability



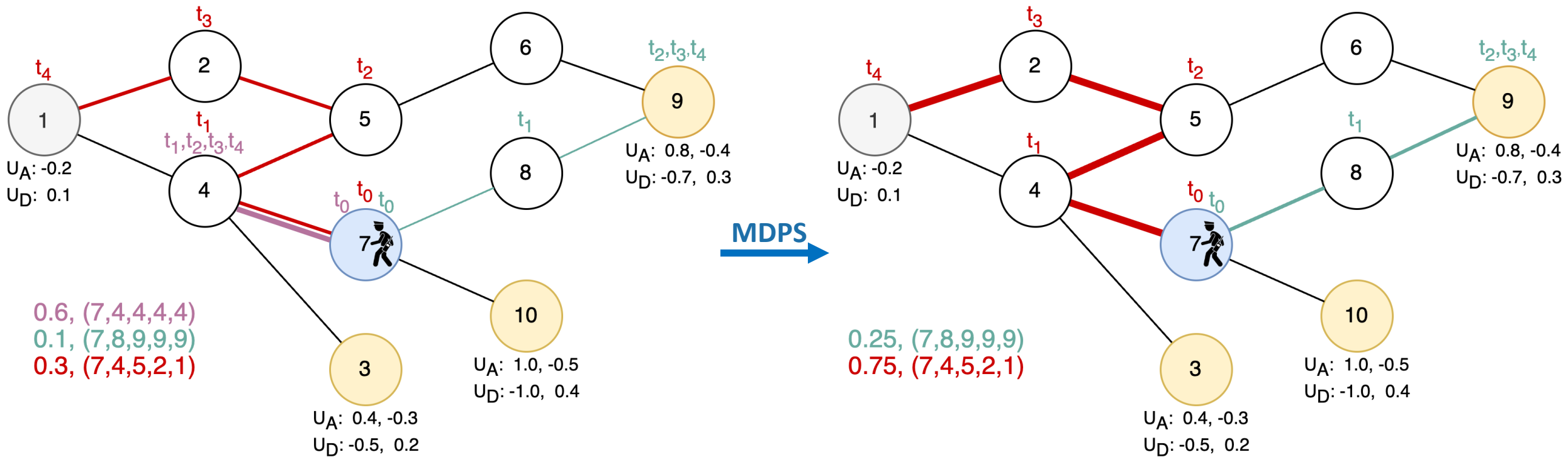
MCP - mutation changes probability - a probability of randomly selected pure strategy is uniformly changed



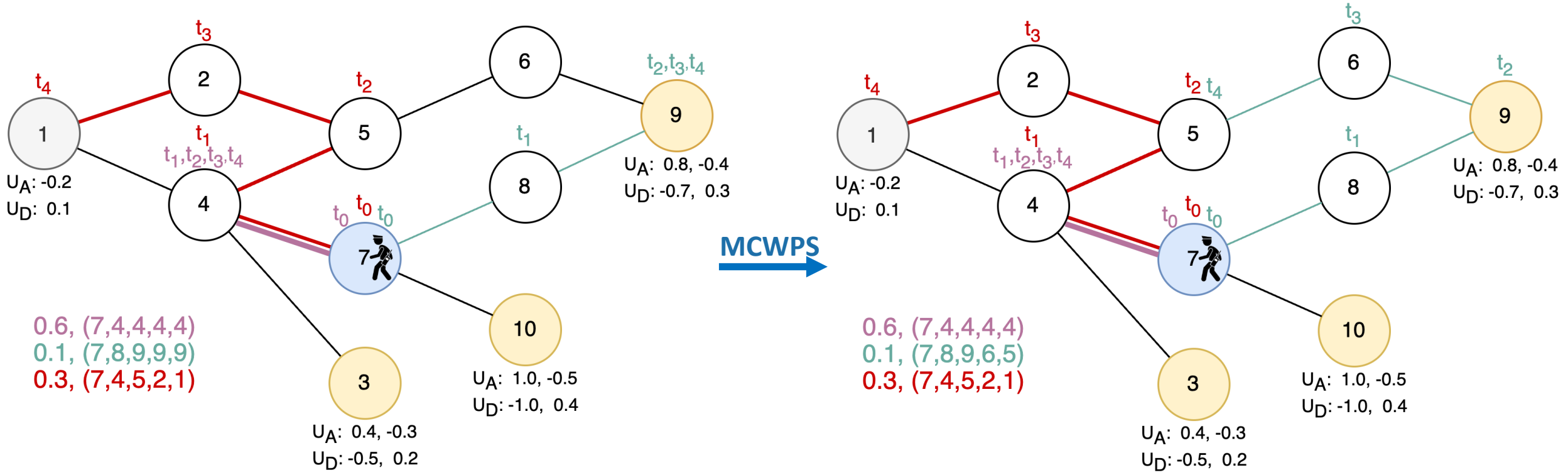
MSP - mutation switches probability - probabilities of two randomly chosen pure strategies are switched



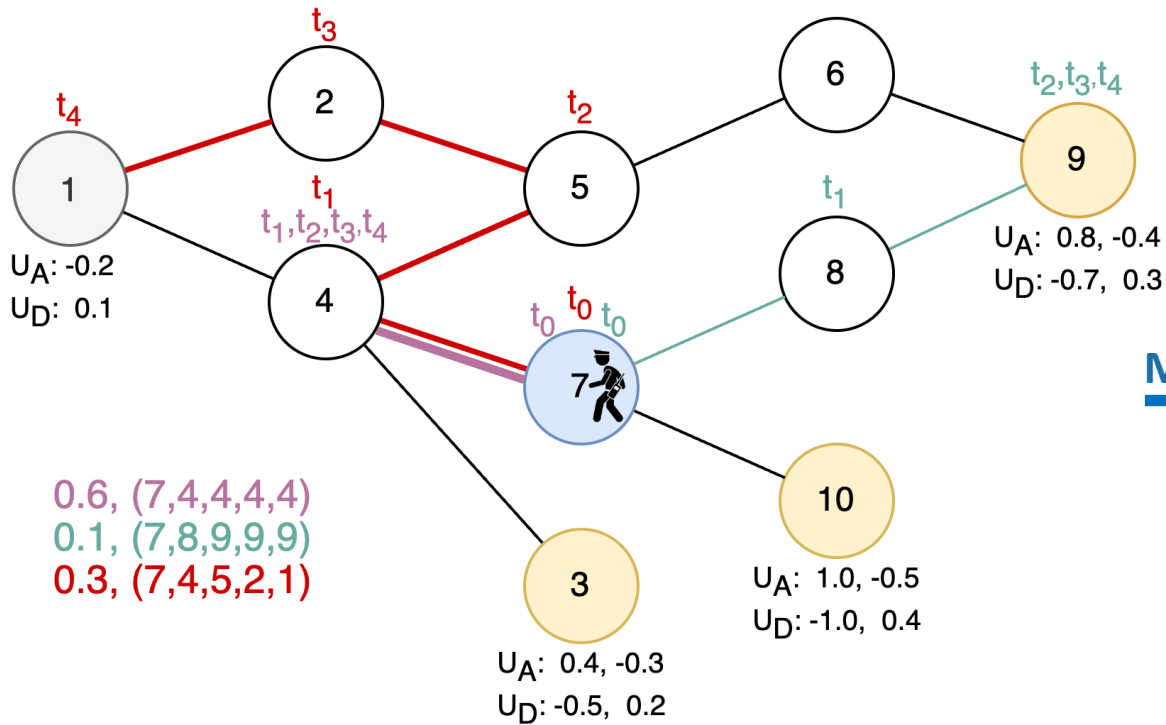
MDPS - mutation deletes pure strategy - a randomly chosen pure strategy is removed



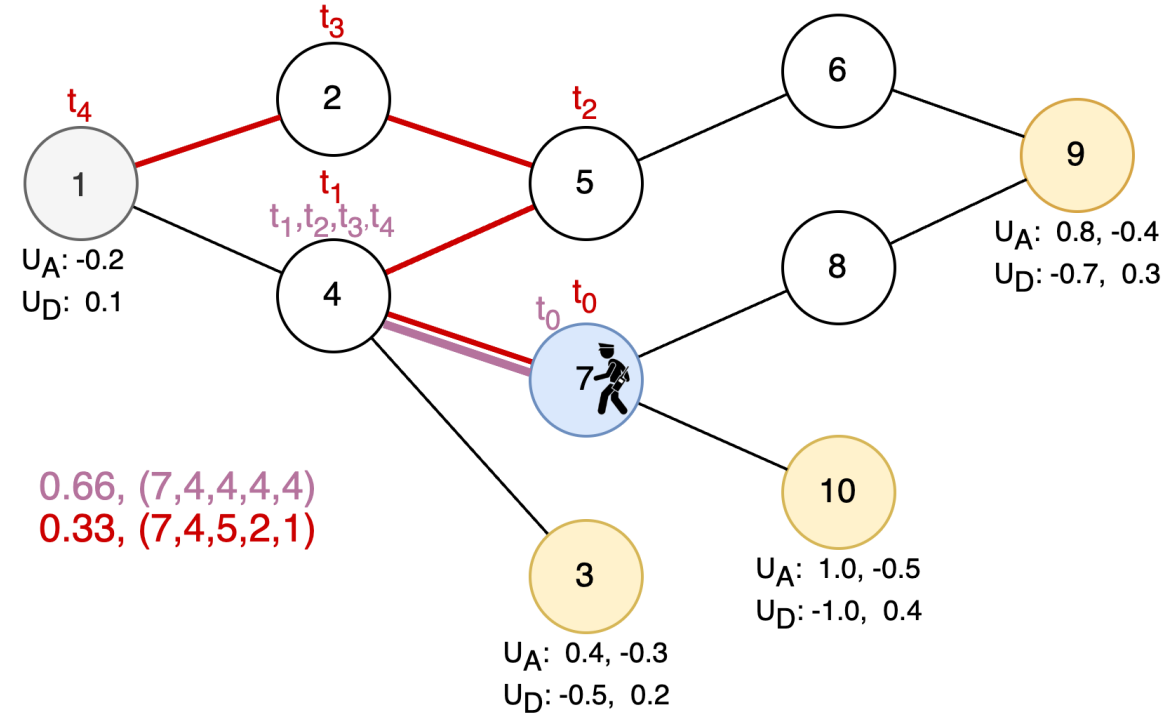
MCWPS - mutation changes the weakest pure strategy - mutation is applied only to a pure strategy with the lowest payoff



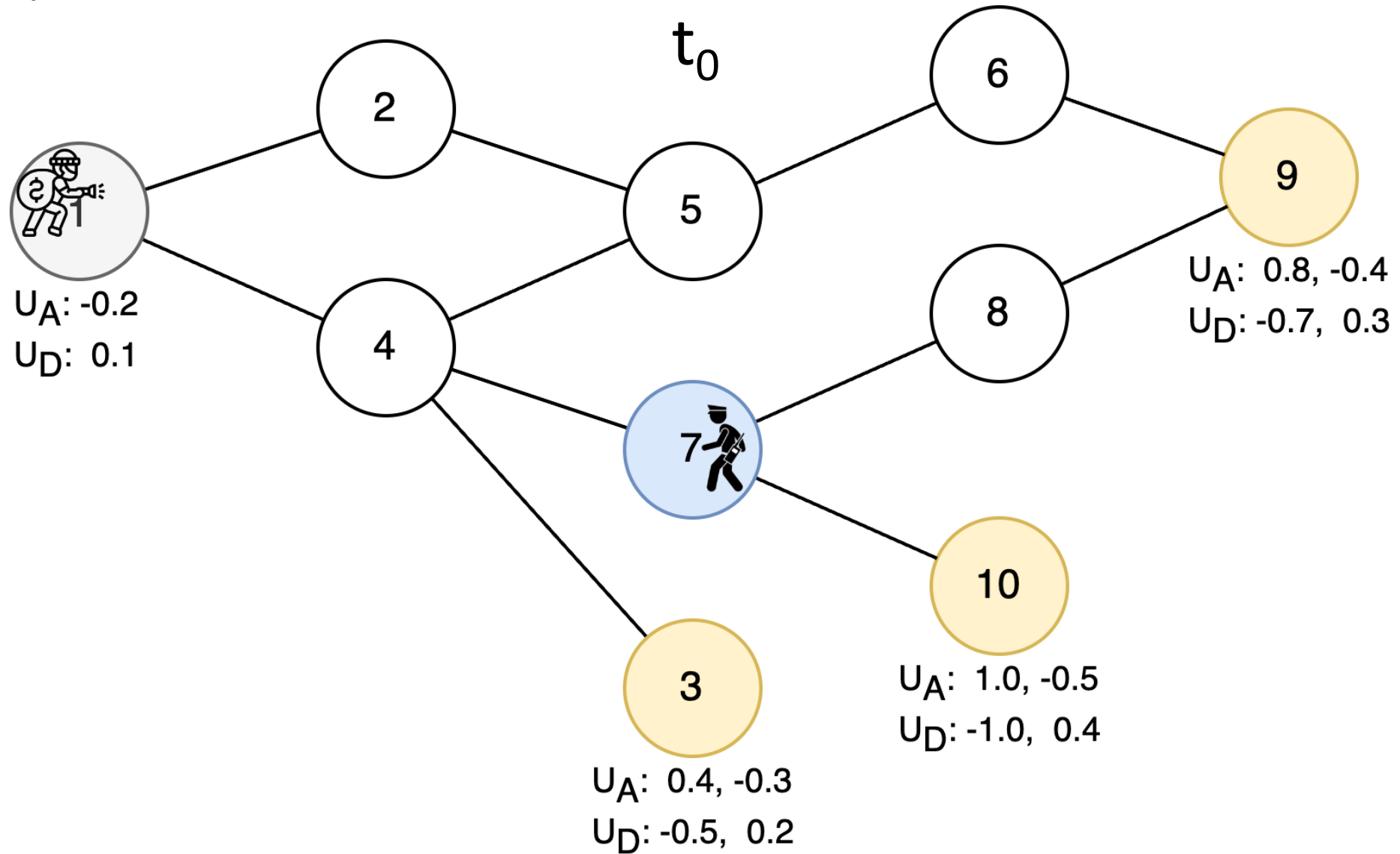
MDWPS - mutation deletes the weakest pure strategy - pure strategy with the lowest payoff is deleted



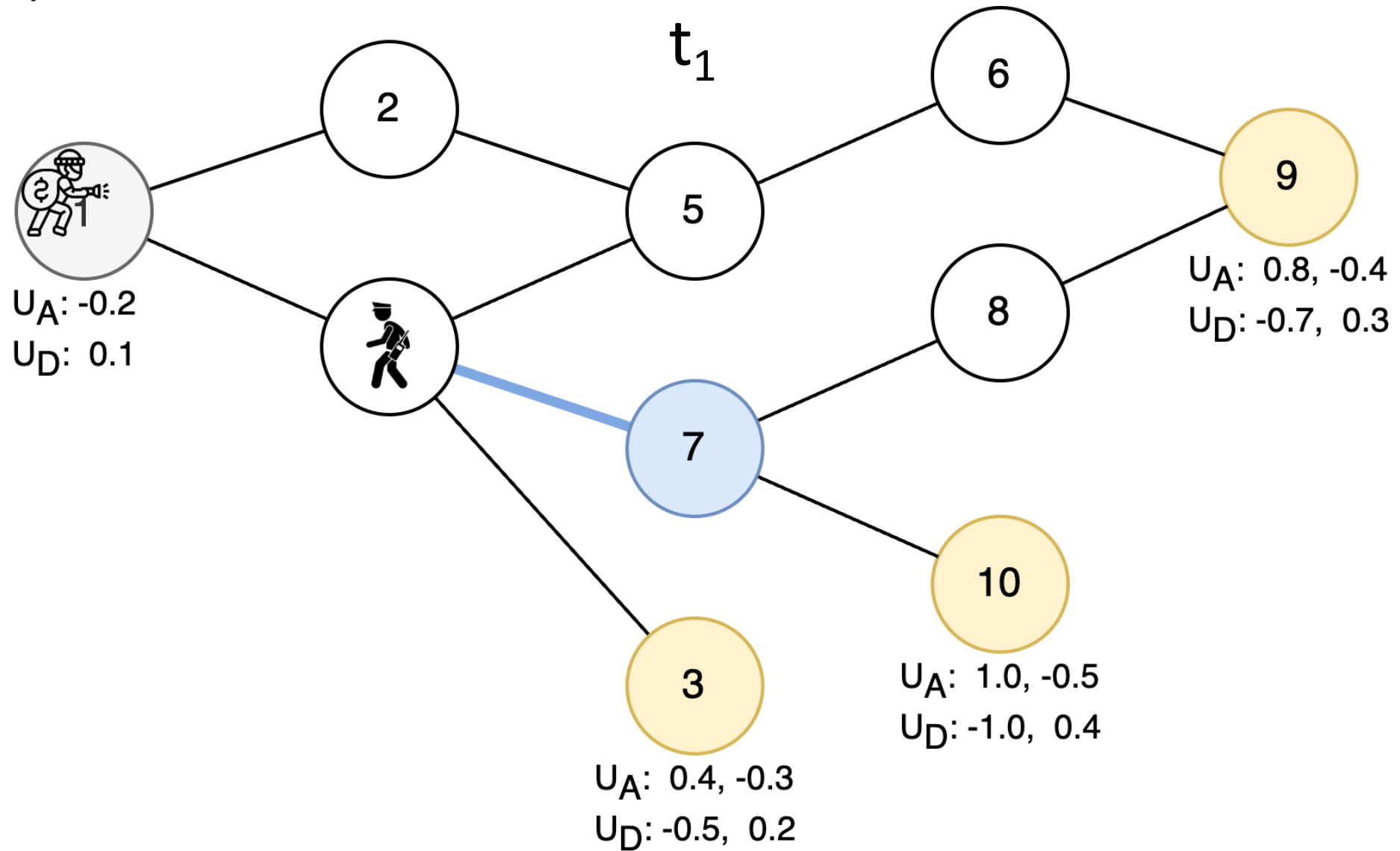
MDWPS →



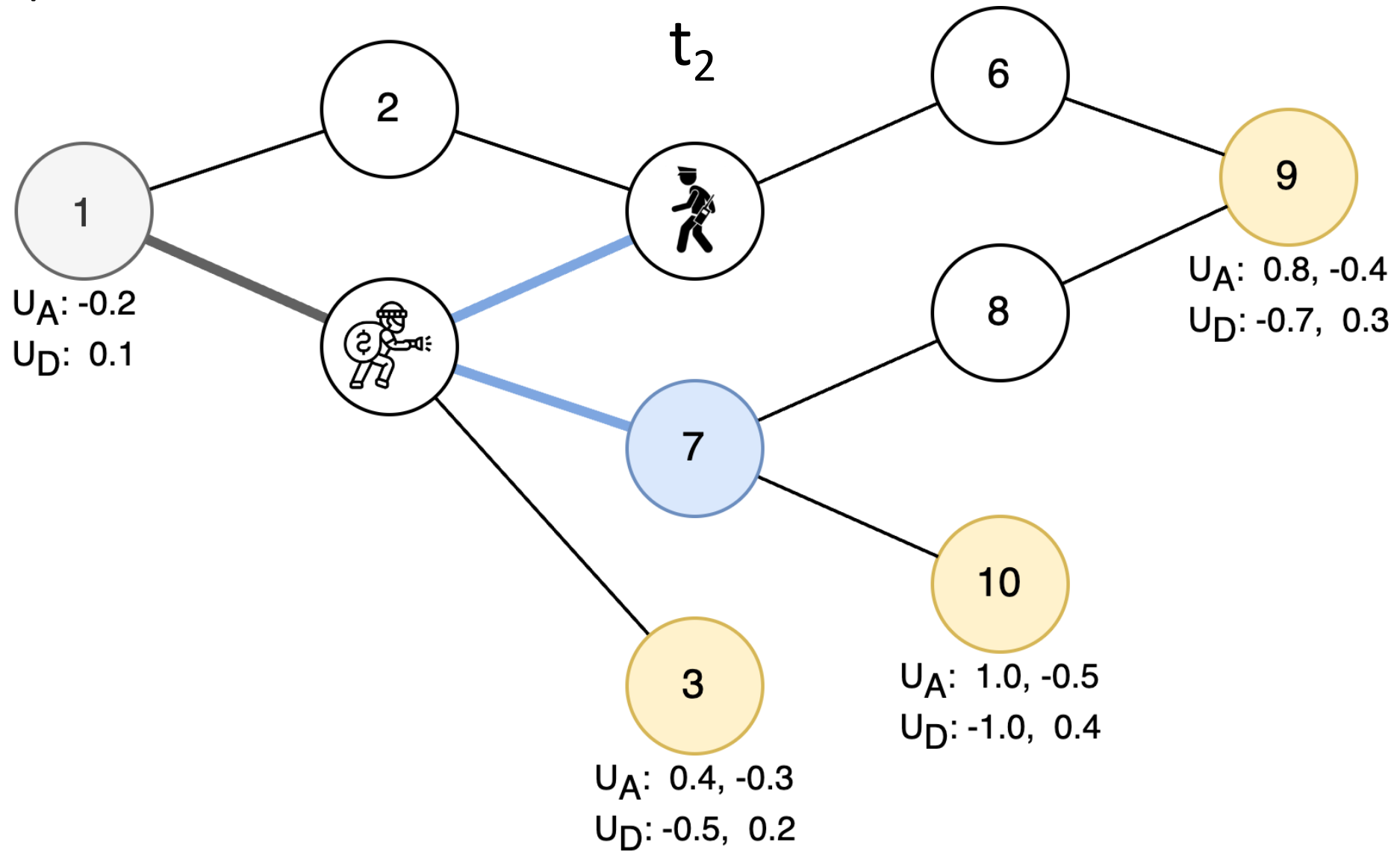
Example – scenario 2



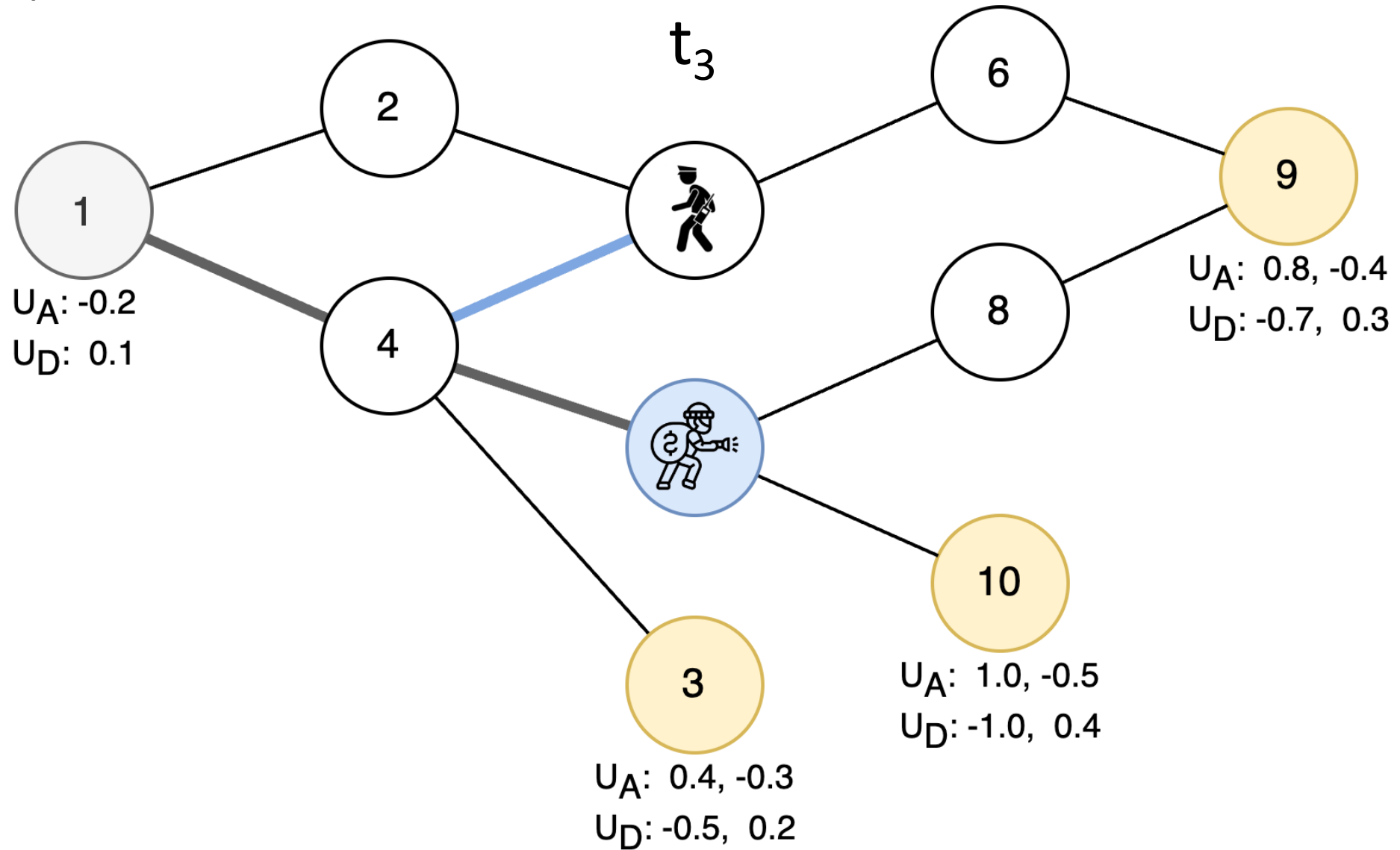
Example – scenario 2



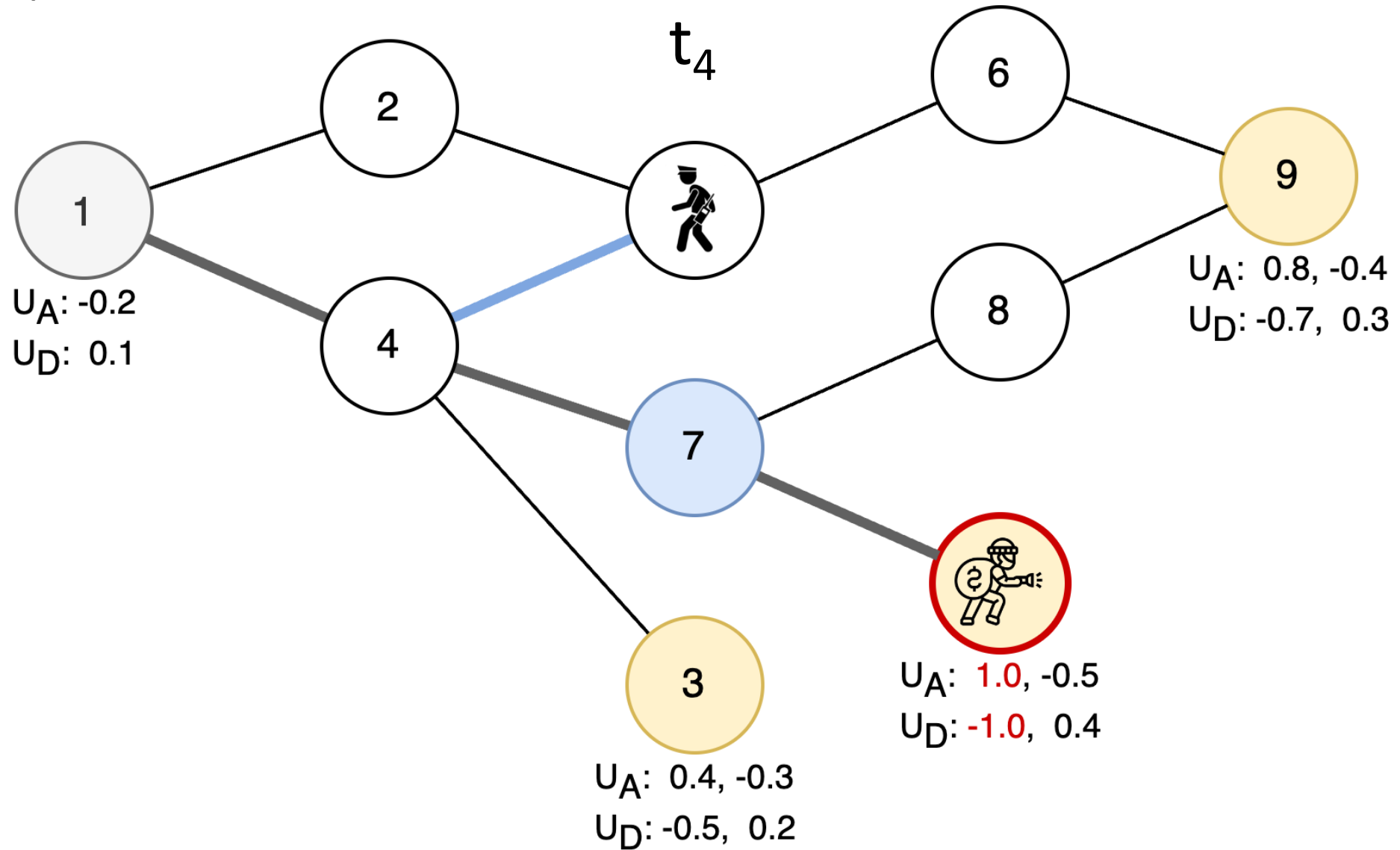
Example – scenario 2



Example – scenario 2



Example – scenario 2



References

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