

Optimized mutation operator in evolutionary approach to Stackelberg Security Games

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Abstract

We introduce several **mutation modifications in Evolutionary Algorithm for finding Strong Stackelberg Equilibrium in sequential Security Games**. The mutation operator used in the state-of-the-art evolutionary method is extended with several greedy optimization techniques. Proposed mutation operators are comprehensively tested on three types of games with different characteristics (totally over **300 test games**). The experimental results show that application of some of the **proposed mutations yields Defender's strategies with higher payoffs**. A trade-off between the results quality and the computation time is also discussed.



Stackelberg Security Games (SSGs)

- Two asymmetrical players: **Defender and Attacker**
- Each game is composed of m time steps.
- Each player chooses an action to be performed in each time step.
- A player's *pure strategy* σ_P ($P \in \{D, A\}$) is a sequence of their actions in consecutive time steps: $\sigma_P = (a_1, a_2, \dots, a_m)$.
- Many real-life applications: e.g. cybersecurity, scheduling canine patrols, protecting Boston Harbor, preventing poaching.

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Defender commits to his/her strategy first.
Attacker, knowing the Defender's strategy, chooses his/her strategy.
Defender always commits to a mixed strategy.

Stackelberg equilibrium: a pair of players' strategies, for which strategy change by any of players leads to his/her result deterioration.

$$(\pi_D^*, R(\pi_D^*)) \in \Pi_D \times \Pi_A$$

$$\pi_D^* = \operatorname{argmax}_{\pi_D \in \Pi_D} U_D(\pi_D, R(\pi_D))$$

$$R(\pi_D) = \operatorname{argmax}_{\pi_A \in \Pi_A} U_A(\pi_D, \pi_A)$$

$G \in \{D, A\}$ - players (Defender, Attacker)

Π_G - a set of player's G all mixed strategies

U_G - payoff of player G

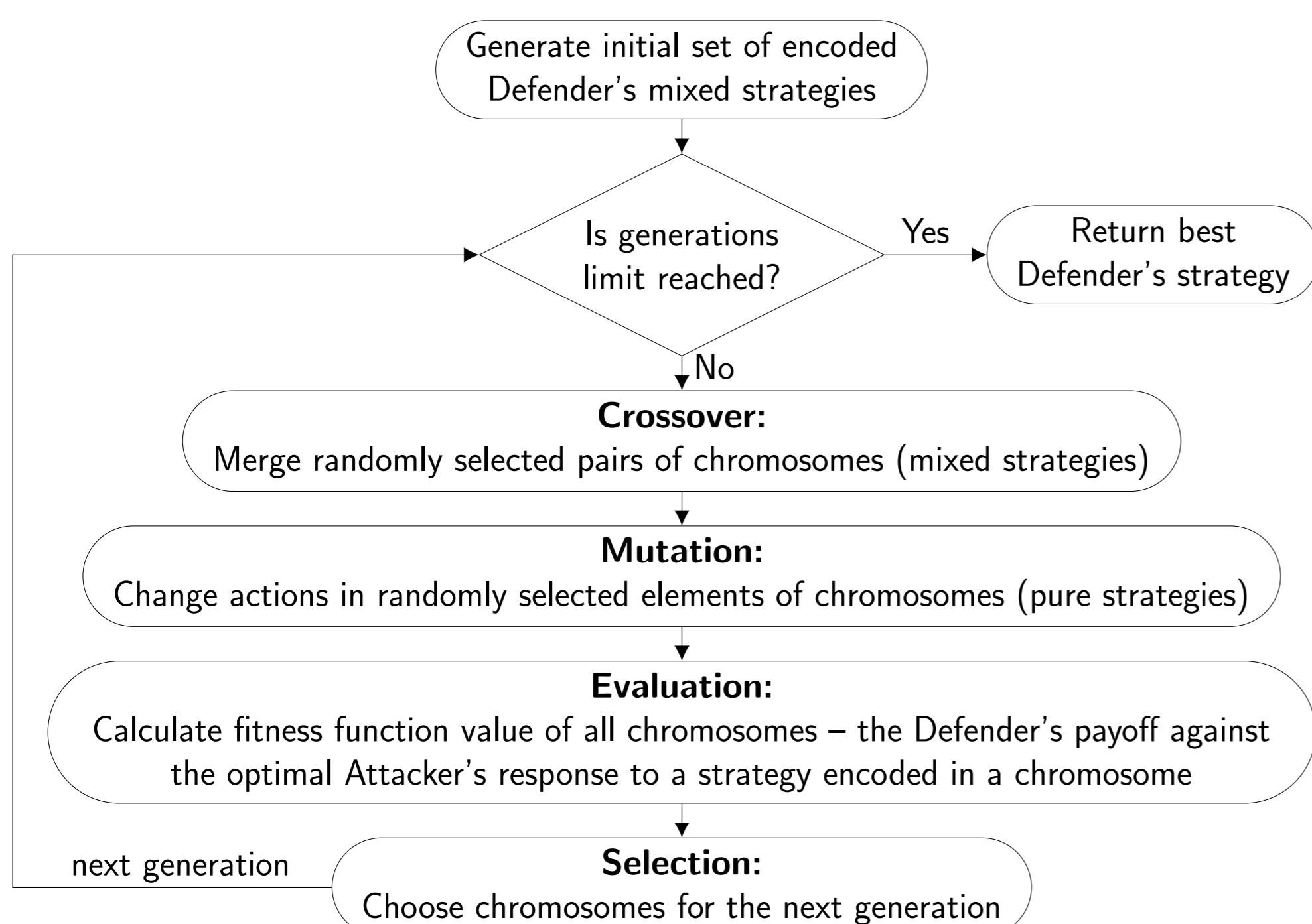


Figure: An overview of the EASG method.

Evolutionary Algorithm for Stackelberg Games (EASG)

EASG [1] aims to **optimize the Defender's payoff** by evolving a population of Defender's mixed strategies. Initially, EASG creates a population of pure Defender's strategies selected at random. The population evolves over successive generations until the stopping criterion is met. Four operations are applied in each generation: crossover, mutation, evaluation, and selection.

Defender's strategy encoding:

$$CH_q = \{(\sigma_1, p_1), \dots, (\sigma_l, p_l)\}, \sum_{i=1}^l p_i = 1$$

Mutation operator randomly selects a pure strategy encoded in the chromosome and modifies it, starting from a randomly selected time step. New actions are drawn from the set of all feasible actions in a given game state.

Mutation enhancements

- EASG_n** - EASG algorithm with repeated mutation.
- MANPS₁, MANPS_n** - *mutation adds new pure strategy* - a uniformly selected pure strategy is added with a uniformly sampled probability.
- MCP₁, MCP_n** - *mutation changes probability* - a probability of randomly selected pure strategy is uniformly changed.
- MSP₁, MSP_n** - *mutation switches probability* - probabilities of two randomly chosen pure strategies are switched.
- MDPS₁, MDPS_n** - *mutation deletes pure strategy* - a randomly chosen pure strategy is removed.
- MCWPS** - *mutation changes the weakest pure strategy* - mutation is applied only to a pure strategy with the lowest payoff.
- MDWPS** - *mutation deletes the weakest pure strategy* - pure strategy with the lowest payoff is deleted.

Results

Table: The average and standard deviation values of the Defender's payoff and the computation time for various mutation operators. The best results are **bolded**. Results that are better than the baseline version of the algorithm (EASG) are underlined. In cases where the difference between the baseline version (EASG) and a given variation is statistically significant (according to the Wilcoxon test with p -value < 0.05), the result is highlighted with a gray background.

	Defender's payoff			Computation time [s]		
	WHG	SEG	FIG	WHG	SEG	FIG
EASG	0.017	0.108	0.031	152	2534	328
EASG _n	0.017	<u>0.135</u>	0.037	1206	21913	3051
MANPS ₁	0.014	0.059	0.031	156	2548	313
MANPS _n	0.016	0.139	<u>0.036</u>	1366	21892	2988
MCP ₁	0.015	0.074	0.030	148	2422	336
MCP _n	0.016	<u>0.131</u>	0.037	1285	22651	3008
MSP ₁	0.013	0.099	0.024	156	2583	316
MSP _n	0.016	0.108	0.037	1332	21447	2931
MDPS ₁	0.013	0.052	0.029	147	2620	313
MDPS _n	0.013	0.053	0.026	1283	22026	2900
MCWPS	0.013	0.046	0.030	148	2612	321
MDWPS	0.008	0.058	0.018	139	2361	299

Conclusions

- Repetition of mutation operation leads to improvement of SSGs outcomes**, though at the expense of significant increase in computation time.
- The proposed modifications offer a **viable alternative to the base EASG formulation** for cases when computational cost is less important.

References

[1] Żychowski A., Mańdziuk J. Evolution of Strategies in Sequential Security Games. In Proceedings of the 20th AAMAS conference, pages 1434-1442. 2021.